

Voronoi algorithm + Design

Dissertation by Jayant Khanuja. 1703 S.I.D

Understanding algorithms in design with reference to Voronoi diagrams

By

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Dissertation

Submitted to the Faculty of the
Interior Design, CEPT University.



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UNDER GRADUATE PROGRAMME IN INTERIOR DESIGN

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**Thesis Title : UNDERSTANDING ALGORITHMS IN DESIGN WITH REFERENCE TO VORONOI
DIAGRAMS**

APPROVAL :

The following study is hereby approved as a creditable work on the approved subject carried out and presented in the manner, sufficiently satisfactory to warrant its acceptance as a pre-requisite to the degree of Bachelor of Interior Design for which it has been submitted.

It is to be understood that by this approval, the undersigned does not endorse or approve the statements made, opinions expressed or conclusion drawn therein, but approves the study only for the purpose for which it has been submitted and satisfies him/her to the requirements laid down in the academic programme.



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2D Voronoi
Centroidal Voronoi Tessellation
3d Voronoi
Other types of Voronoi

Chapter two.
Voronoi Geometry in nature

Dragonfly wing
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Packing
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C_Wall

Smart Cloud

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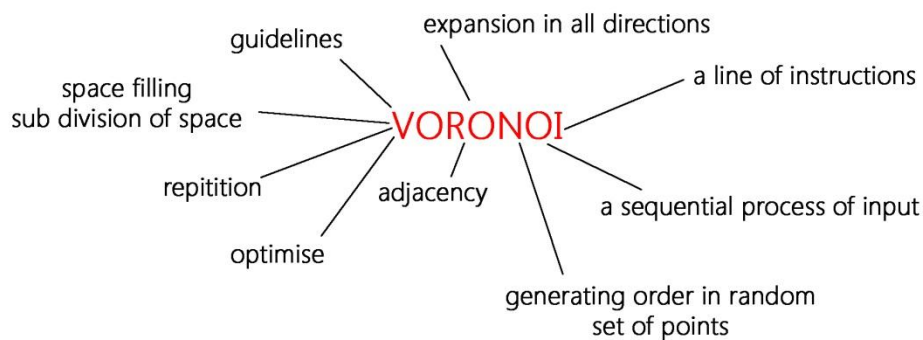
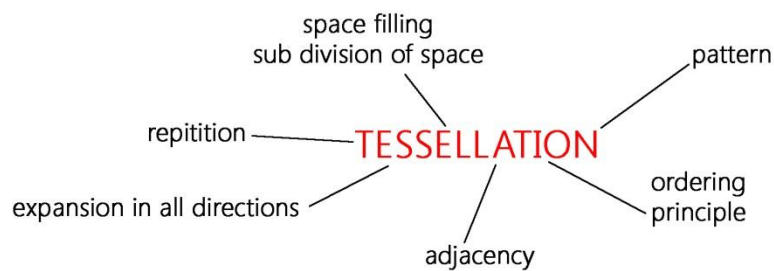
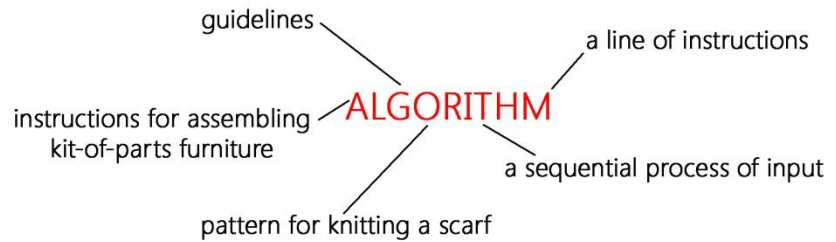
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Introduction



"There is a lot more to structure than strict post and beam. Slabs may fold and act as lines of vertical strengths, beams may bifurcate and change shape, columns can serve as beams, the ingredients are all there to evolve form in fascinating ways. The challenge is to make structure the new discipline in a re-examination of space."

CECIL BALMOND, LONDON, January 2002

ALGORITHM in DESIGN

A code – often called an algorithm, procedure, or program – defines a specific process with enough detail to allow the instructions to be followed.

Software influences all aspects of contemporary design and visual culture.

Algorithms offer a non-technological implication in design. They break down the elusive and sometimes problematic phenomena of shape. Shapes are never unwilled figures. Deep within them is a struggle between the predilections of the designer and the inherent properties of the geometries encountered. The algorithm mediates these two, acting as a kind of solvent to liquefy them and create the potential for crystallization.¹

TESSELLATION in DESIGN

Tessellations – the orderly basis of subdivision of space.

Systems can be envisaged which consist of some minimum inventory of component types which can be alternatively combined to yield a great diversity of efficient structural form. These are called minimum inventory/maximum diversity systems.

It is well known that, of all plane figures, the circle encloses the greatest area of surface for a given circumference. Likewise, of any three-dimensional shape the sphere matches the greatest volume to the least surface area.

*Do the circle and the sphere remain the most economical possibilities when we are concerned with the many-celled partitioning of space?*²

VORONOI ALGORITHM as a tool for TESSELLATION/TILING as a tool for DESIGN

Principles, such as minimal surfaces, repetitive tiling or snowflake formations can be used as inspiration to develop abstract diagrams that in turn can be refined and enriched with architectural information to become prototypical organizational models for buildings. Illustrations of mathematical concepts like knots or visualization of algorithms (voronoi) provide another source of inspiration. They open up the possibility to think about other worlds, environments and building concepts, besides the platonic solids, Cartesian grids and equally spaced grid systems that dominated design process for so many centuries.³

¹ Tooling , Pamphlet Architecture – Aranda/Lasch.

² Structure in Nature is a strategy for design – Pearce, Peter

³ Mathknow Mathematics, Applied Sciences and Real life.

All visual patterns and tessellations at their core are composed of algorithms. Even centuries old patterns, such as Scottish tartans, follow strict compositional rules that are capable of being encoded into software. Writing code is an exciting way to approach tessellations/tiling.⁴

Digital Methods and Design

The connection of design and computer science generates manifold possibilities for the development of new products and services. The computer has become a comprehensive and dynamic medium in developing and realizing spatial concepts. Exploiting this potential requires the ability to use the computer as an interactive instrument and use its artificial intelligence as an expansion of possibilities. Developments over the past 20 years in computer aided architectural design and production show the steadily growing influence of digital media on the work of designers. Processes for developing architectural concepts and formal designs, and even the way designs are perceived, have evolved considerably through the implementation of computer technology.

Computer-Aided Design (CAD) enables the designers to generate complex spatial geometries. The new freedom leads often to an increasing number of arbitrary free-from-shapes that promise spectacular spaces on the computer screen but often fail in the final built project. The lack of traceable relations between the different aspects and elements of designs result in complicated, rather than complex structures. Complexity though, as an intelligent connection of elements to form an integral overall structure, is one of the main characteristics of designs. In this respect geometry delivers a perfect toolbox of various principles to organize a flexible pattern that can be transformed, manipulated and expanded in the further process. The knowledge of geometric rules and principles is the essential condition to develop a solid base for the design- and realization-process of complex spatial concepts. Starting from the initial form-finding, geometry guides the project through the optimization of the shape, the integration of various elements and the implementation of parameters regarding manufacturing and assembling in the realization-process.

The computer now enables that which once divided the designers: the aspect to build. The understanding of Geometry plays a major role in the application of the new digital techniques by architects. Sometimes, it is used as an inspirational concept, but more and more often a deep understanding of geometrical relationships is the key for parametrical optimization and associative modeling techniques. These design processes trigger a different notion of form as the result of a process rather than the idea of a single designer.

“The dynamics of line and surface define the templates that create architecture and organizations. In the making of form it is not only shape that counts but the rules and interior logic by which such contours are derived” – Cecil Balmond.

Aim and Objectives

Study will focus on understanding the definition, construction and implementation of an algorithm in design.

Objective of the study is to understanding the complexity of Voronoi Algorithm through diagrams and not by mathematical calculations and how it can be implemented in a design process to give outputs based on the aspects of mass customization, material optimization and aesthetics.

Scope of Research

This research will help to provide a better perspective on how digital tools can be used in a much more technical as well as creative manner. Time and options in a particular design are two important factors in the process of designing. Thus this will also provide a glimpse of showing that how computations can act as a catalyst for a design process.

⁴ Form+Code In design, art and architecture. – Casey Reas, Chandler McWilliams, LUST.

Limitations

All case Studies are second hand data though most of it is taken directly from the original source.

Methodology and Framework –

Methodology – In this dissertation firstly Voronoi algorithm is introduced and analyzed diagrammatically Later on the methodology goes according to this chart

Approach in design

Algorithms as a tool in design

A broad perspective understanding algorithmic designs

Tessellations as a resultant of algorithms

Narrowing down the approach with the filter of resultant as just tessellations

Voronoi Algorithm to generate tessellations

Further narrowing the study w.r.t selection of voronoi algorithm

After theoretical framework there is analytical framework which includes all the case studies. Lastly an investigation into digital methods and production is included and then there is conclusion.

Framework

The study is divided into two parts

- Theoretical Framework
- Analytical Framework

Theoretical Framework

This section consists of all the theory which is mostly represented through diagrams for the technical part.

Chapter one – Basics of Voronoi geometry
Chapter two – Existence of Voronoi Geometry in nature.
Chapter four - Close Packing of Geometry(Tessellation or Tiling).
Chapter Seven – Investigation into digital methods and manufacturing.
Chapter Eight – Conclusion.

Analytical Framework

It contains od all the case studies which are divided into these categories

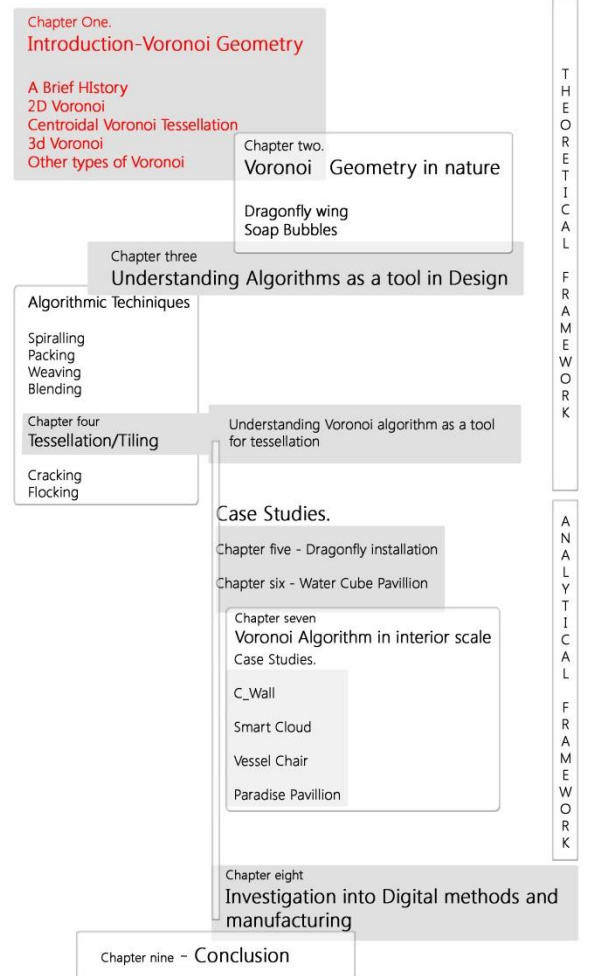
- Design Metaphor
- Strategy
- Material Behavior
- Structural Performance
- Installation Methodology

Chapter three- Case Study Dragonfly installation.
Chapter five – Case Study Water Cube Pavilion.
Chapter Six – Application of Voronoi Geometry in interior elements.

General Bibliography

- Form+Code In design, art and architecture. – Casey Reas, Chandler McWilliams, LUST.
- Tooling, Pamphlet Architecture – Aranda/Lasch.
- Structure in Nature is a strategy for design – Pearce, Peter
- Mathknow Mathematics, Applied Sciences and Real life.
- Balmond, Cecil. Informal. Munich, Berlin: Prestel, 2002.

Acknowledgments



Geometry

1.1 -Voronoi diagrams – A brief History

According to Okabe in his book Spatial Tessellations [2], some of the first uses of the Voronoi diagram were recorded as early as the 17th century by the well-known philosopher Descartes.

In his works, Descartes used weighted Voronoi diagrams to explain how matter is distributed throughout the solar system. (Figure 1), a diagram of certain points on the Euclidean plane separated by lines that are equidistant from the closest two such points, is an example of Descartes' work.

As seen in nature many natural structures closely resemble Voronoi diagrams and it seems unlikely that such structures would have gone unnoticed by early scientists and observant laymen alike.

The first undisputed comprehensive presentations of the concept appeared in the work of Peter Gustav Lejeune

Dirichlet(1805-1859) and Georgy Fedoseevich Voronoy (Georges Voronoi) (1868-1908) who, in their studies on positive quadric forms (Dirichlet, 1850; Voronoi, 1907, 1908, 1909) considered a special form of Voronoi diagram.

Dirichlet treated two-and three dimensional cases whereas Voronoi examined the general m-dimensional case. Their concern was with the distribution of points with integer coordinates that give minima of the values of a given quadric form.

Although Georgy Fedoseevich Voronoi (1868-1908) was not the first to study this special kind of decomposition of a metric space, it bears his name in honour of the advancements he made in the theory. He not only unequivocally defined the Voronoi diagram. In honour of Dirichlet, who also made significant advances to Voronoi theory, Voronoi diagrams are sometimes also referred to as Dirichlet tessellations.

Voronoi diagrams provide notions of space in connections and separations, primordial architectural characteristics as described by Georg Simmel in the essay *Bridge and Door* ⁵

"Only to humanity, in contrast to nature, has the right to connect and separate been granted, and in the distinctive manner that one of these activities is always the presupposition of the other.[...] The forms that dominate the dynamics of our lives are thus transferred by bridge and door into the fixed permanence of visible action."

Given that the initial developments of the Voronoi diagram concept involved sets of points which were regularly placed in space, it is not surprising that some of its first applications were in crystallography. Work in this area began in the late nineteenth and early twentieth centuries and was dominated by German and Russian researchers. At this time Voronoi regions were known by a number of names.⁶

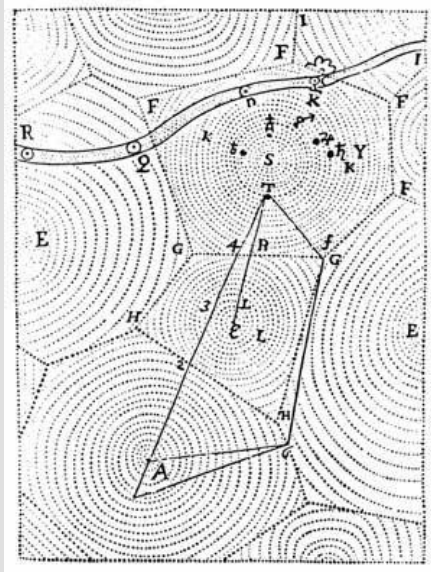


Figure 1 - Descarte's Voronoi diagram.

⁵ Virtual Systems and multimedia, 13th international conference.

⁶ Spatial Tessellations – Concept and Applications of Voronoi Diagrams-Second edition
Atsuyuki Okabe, Barry Boots. Kokichi Sugihara, Sung Nok Chiu.

Definition of Voronoi Tessellations.

In the 2-D case, a Voronoi diagram is a partition of the plane into n convex polygons. Each partition contains one generator such that every point in the partition is closer to its own generator than any other generator.⁷

- Voronoi generators - the set of point belonging to the closed set in a Voronoi diagram that is used to form the distinct Voronoi regions.
- A Voronoi Region - the convex area (in Euclidean space) that contains every point closest to a generator with respect to all the other generators.
- A Voronoi Edge is the line, half line, or line segment that corresponds to the points that connect exactly half way between two Voronoi Generators.
- A Voronoi vertex is the intersection of three (or more, depending on the restrictions of the given Voronoi Diagram) Voronoi Edges.

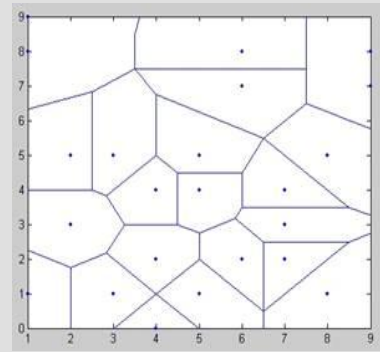


Diagram 1 – Voronoi diagram of random set of points

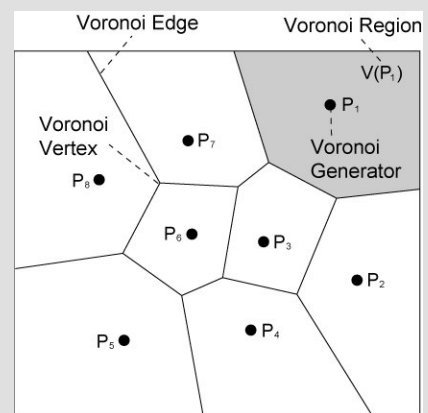


Diagram 2 -Graphical Definitions of Voronoi Pieces

⁷ Spatial Tessellations – Concept and Applications of Voronoi Diagrams-Second edition
Atsuyuki Okabe, Barry Boots. Kokichi Sugihara, Sung Nok Chiu.

Definition of Delaunay triangles –

- The Delaunay triangulation of a point set is a collection of edges satisfying an "empty circle" property: for each edge we can find a circle containing the edge's endpoints but not containing any other points. Fig (last two)

After the triangulation, the voronoi lines are calculated from the Delaunay triangles.

- In mathematics and computational geometry a Delaunay triangulation for a set P of points in the plane is a triangulation $DT(P)$ such that no point in P is inside the circumcircle of any triangle in $DT(P)$.
- Delaunay triangulations maximize the minimum angle of all the angles of the triangles in the triangulation.
- They tend to avoid skinny triangles.
- The triangulation was invented by Boris Delaunay in 1934

Relationship of Delaunay triangulation with Voronoi diagrams.

The Delaunay triangulation of a discrete point set P in general position corresponds to the dual graph of the Voronoi tessellation for P . Special cases include the existence of three points on a line and four points on circle.

Diagram 5a

The Delaunay triangulation with all the circumcircles and their centres (in red)

Diagram 5b

Connecting the centers of the circumcircles produces the Voronoi diagram (in red).

Diagram 3a -original point set as base for construction of Voronoi.

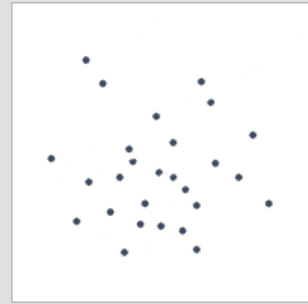


Diagram 3b -Delaunay triangulation on the original point set.

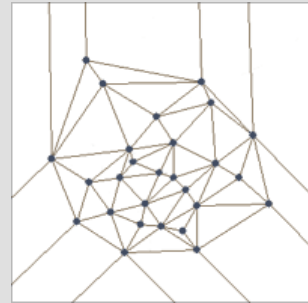


Diagram 3c -Voronoi diagram built from Delaunay triangulation.

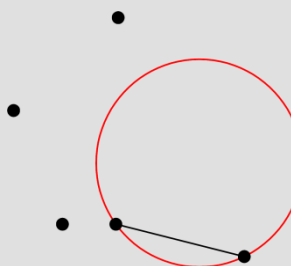
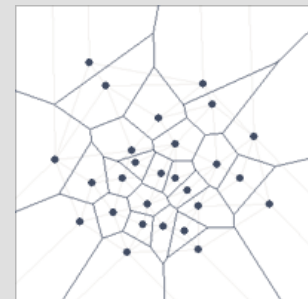


Diagram 4a - Correct configuration of points, as no point lies inside the circle

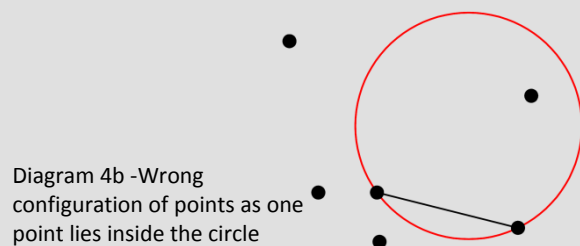


Diagram 4b -Wrong configuration of points as one point lies inside the circle

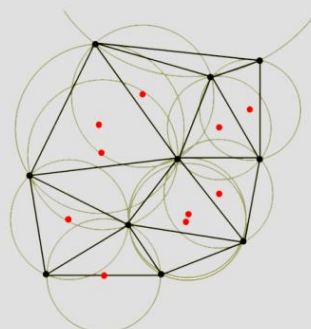


Diagram 5a.

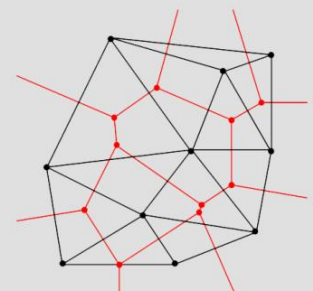


Diagram 5b.

Construction of voronoi geometry

Basic Elements for its construction-

Input Parameters

- Set of points – can be random or ordered.
- Bounding Box – defining the extents of the geometry.

Reference geometry for its construction-

- Circles – drawn from 3 to 4 nearest set of points
- Perpendicular lines – drawn from the midpoint of the line connecting two nearest points.

Output geometry

- Set of lines in case of 2 dimensional constructions.
- Set of planes in case of 3 dimensional construction

Construction Process can be sequential

Figure a – Generated through voronoi plugin in rhinoceros software.

Figure b – Drafted manually in rhinoceros software without using the plug in.

- Each vertex consists of 3 edges.
- Mostly 5 edges exist for each cell.
- Edges on the outer phase derived by connecting centre (circle) to midpoint (edge).

Figure c – Generated in rhinoceros through grasshopper plugin. This one gives the possibility of defining a bounding box of required size and volume.

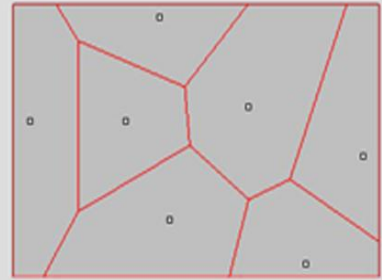


Diagram 6a – Construction of Voronoi geometry on 7 points arranged randomly, generated in rhinoceros software but a pointsetreconstruction tools plugin.

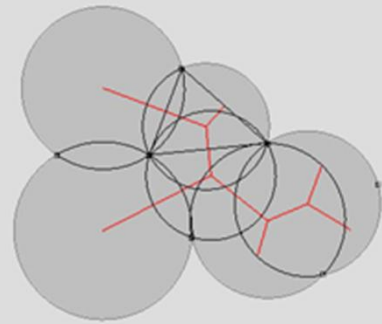
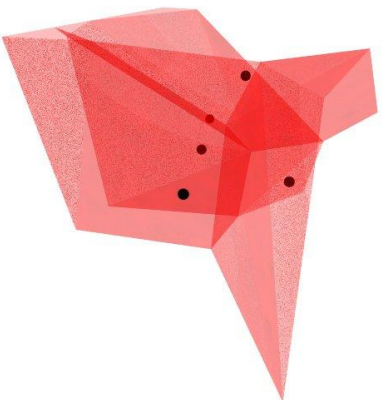


Diagram 6b – Construction of Voronoi geometry, drafted manually.



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Diagram 6c – Construction of 3d Voronoi in rhinoceros software through grasshopper plugin.

Computing the voronoi tessellation.

There has been developed a huge variety of methods to compute Voronoi diagrams, but basically they can be divided into 2 categories:

- algebraic methods, using purely analytical methods.
- geometric methods, using a geometrical model of the point properties.

Examples of the two calculation methods are shown in the following:

Algebraic method.

This method is built basically on the Delaunay triangulation of the space occupied by the points.

Figure 14 -

Two sites from a perpendicular bisector.

In this case Voronoi Diagram is a line that extends infinitely in both directions, and the two half planes on either side.

Figure 15 -

Collinear sites form a series of parallel lines

Figure 16 - (Hansen)

Non-collinear sites form Voronoi half lines that meet at a vertex

A vertex has degree ≥ 3

V - A Voronoi vertex is the center of an empty circle touching 3 or more sites.

Figure 17 -

For 2D Voronoi diagrams four or more non - collinear sites are required to create a bounded Voronoi cell



Diagram 7a – Resultant diagram from 2 points.

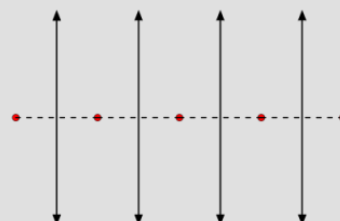


Diagram 7b – Resultant diagram from points arranged linearly.

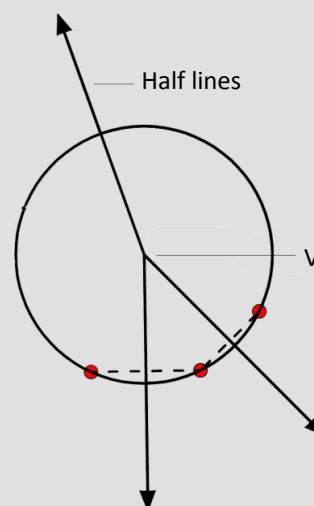


Diagram 7c – Resultant diagram from points arranged randomly.

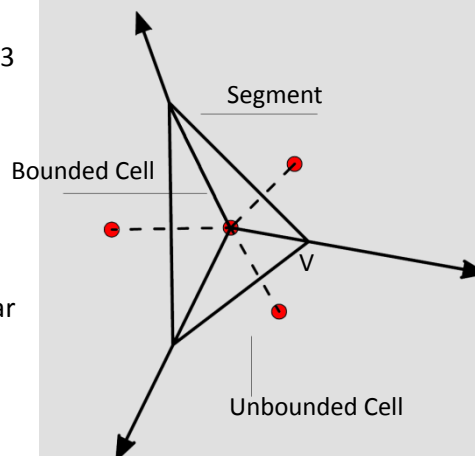


Diagram 7d – Resultant Voronoi diagram from points arranged randomly.

Geometric Method

The mathematician Steven Fortune has developed a geometrical method to compute the Voronoi lines.

He used the sweep-line strategy implemented by many algorithms of computer graphics. He puts cones above all points in the plane. These cones get scanned by a scanning plane slanted by 45° . The intersection of the cones with the plane produces parabolas, which, projected onto the horizontal plane, produce the Voronoi lines.

His method is widely used in computation, because of its effectiveness.⁸

• Fortune's Algorithm:

Voronoi diagram constructed as horizontal line sweeps the set of sites from top to bottom.

Due to this Incremental construction:

- maintains portion of diagram which cannot change due to sites below sweep line,
- keeps track of incremental changes for each site (and Voronoi vertex) it "sweeps"

Beach line

The set of parabolic arcs form a beach-line that bounds the locus of all such points

Beach line properties

- Voronoi edges are traced by the break points as the sweep line moves down.
- Emergence of a new break point(s) (from formation of a new arc or a fusion of two existing break points) identifies a new edge
- Voronoi vertices are identified when two break points meet (fuse).⁹

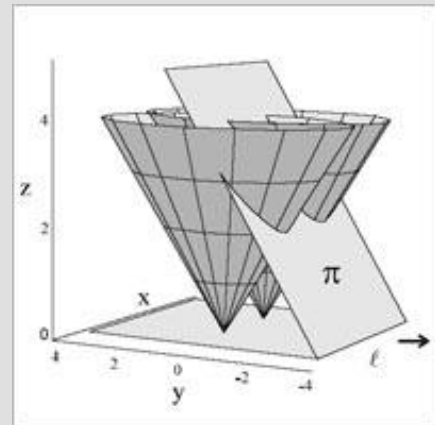


Figure 2– Sweep line strategy to generate Voronoi diagrams.

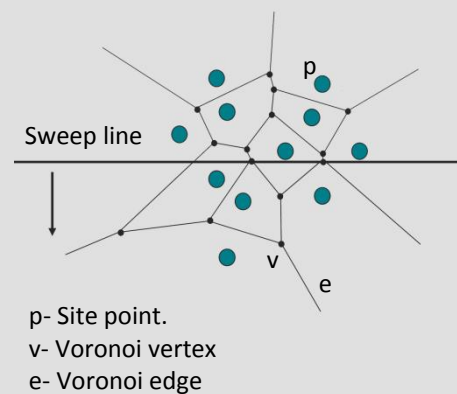


Diagram 8– Resultant diagram from sweep line method.

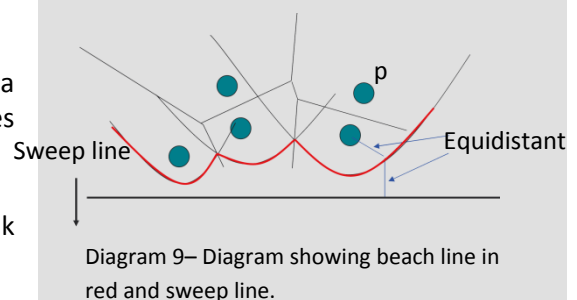


Diagram 9– Diagram showing beach line in red and sweep line.

⁸ V.L Hansen, Geometry in Nature, A K Peters, Ltd, Wellesley, Massachusetts, US.

⁹ Lecture by Allen Miu - PhD Candidate

MIT Computer Science and Artificial Intelligence Laboratory (CSAIL) Sept 30 , 2003

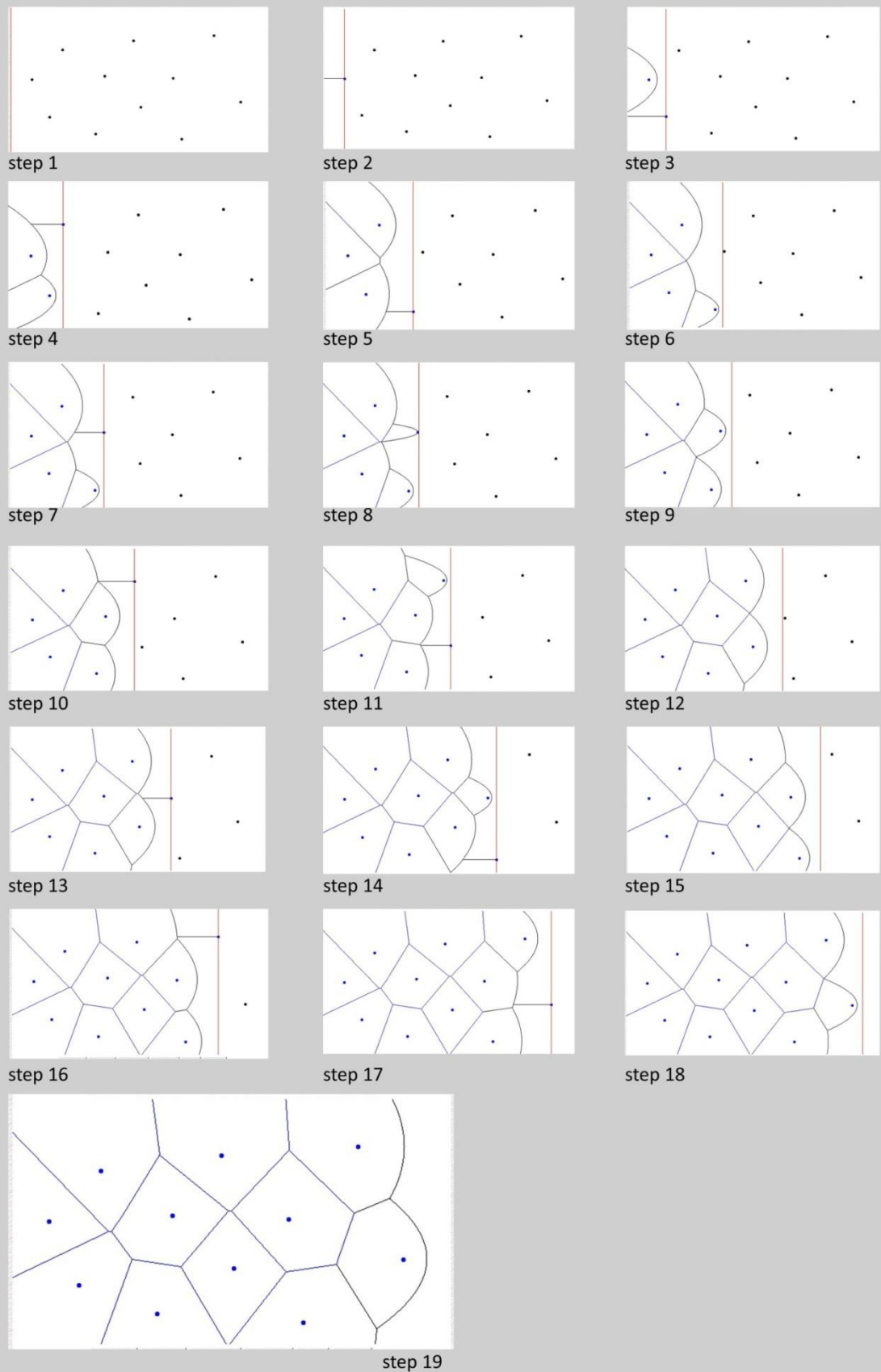


Chart 1– Step wise generation on Voronoi diagram through Sweep line methods.¹⁰

¹⁰ <http://www.diku.dk/hjemmesider/studerende/duff/Fortune/> - Java Applet Screenshots.

Properties of Voronoi Diagrams.

Property 1- If the Voronoi polygon for the generator point p_i contains the point p , then p_i is the closest generator point from p . This is a direct result of the definition of a Voronoi tessellation.

Property 2- Given the set of generators $P = \{p_1 \dots p_n\}$ belongs to a plane R , the Voronoi diagram of P is a unique tessellation of plane R .

Property 3- (The Non-Cocircularity Assumption) Given the set of generators $P = \{p_1, \dots, p_n\} \subset \mathbb{R}^2$ (where $4 \leq n < \infty$), there can be no circle C , centered at a vertex, that contains more than 3 generator points on its circumference while having no other generator inside the area of the circle.

Property 4- For any vertex (q_i) in a Voronoi diagram, there exists a circle (C_i) that passes through at least three generator points, with no generator points in its interior. In the case of the Non-Circularity Assumption, this simplifies to exactly three generator points on the edge of the circle. This circle happens to be the largest empty circle centered at the vertex q_i .

DESIGN and 2D Voronoi:

For a better understanding of 2D Voronoi in design, here design process is seen with a simple categorization in three parts.

1. Design Programme.
2. Concept.
3. Context.

1. Design Programme.

For the purpose of understanding 2D Voronoi, here design programme is understood as a relationship of activities. Design programme can be definitive as well as generative.

A definitive design programme: It is defined here as a programme which has to deal with a much defined set of activities. For such programmes there is a very complex set of permutation and combination when the activities are defined. Such combinations deal with very tangible and functional outputs.

A generative design programme: Such design process has a more give and take with the context; it is a generative process in terms of defining activities and creating an experience.

Design Programme is always a combination of both, a definitive as well as a generative approach is taken into consideration according to design scope and requirement.

Such categorization is done here to have an understanding for the application of this tool (2D Voronoi).

2. Concept.

A design concept is understood here simply as an idea for design and as a solution to any design problem.

Here for the understanding of application of Voronoi design concept is looked upon simply as an initiator and as a problem solver.

An initiator approach – In such approach the design concept or idea acts as a guiding factor throughout the design process.

A problem solver - Such approach is defined as a specific task oriented problem.

3. Context.

For any design process, context acts as an element that gives an identity to the design. Context provides with a varied set of tangible and intangible parameters which can be used as a guiding factor in design process.

When and Why 2D Voronoi?

Application of 2D Voronoi in any design process depends on numerous set of realisations and requirements. For a basic understanding, here its application is understood with reference to the simple categorization done earlier for design process.

Design Programme and 2D Voronoi

If a very definitive programme approach is taken into consideration, 2D Voronoi can be really helpful in terms of deciding layout and zoning. It can help inter relate the required functions and can generate a network and a very specific demarcation. It can be a useful tool in terms of defining activities. Such generated network can be later modified according to requirement.

In a generative programmatic approach the input parameters can be various intangible factors based on the concept and context. For such an approach the output looks the same as a network, but the perception of such network is a lot different. It is looked upon as area and volumes and not in terms of activities.

For any design programme both these programmatic approaches are equally important. The benefit of using such a computational tool is that it gives a lot of options, the network can be taken into iterations, and it also creates a lot of scope for diagramming.

Concept and 2D Voronoi:

An initiator approach: When concept or idea is looked upon and understood in such a fashion, a computational tool such as 2D Voronoi can be really helpful in terms of combining tangible and intangible design variables.

Such a computational tool does not take the input in terms of numbers but it takes it in as entities thus any variable can be fed in and defined as an entity.

2D Voronoi not only allows interrelating design variables but it also helps as a guiding factor in terms of modifications.

A Problem solver : For a very task based problem, such computational tool can help in giving a very defined output, in such a case it is helpful because of its iterative approach and also it not only solves the required problem but it also helps by giving options.

Context and 2D Voronoi:

While dealing with context such a computational tool is useful because as earlier mentioned it helps creating a relationship between tangible and intangible design variables. The resultant of 2D Voronoi is a very flexible output and thus it can be modified and moulded into various required variables.

For such a pattern there is no start or end point, it functions purely on the input data and thus while dealing with context it does not apply its own defined boundaries.

Centroidal Voronoi Tessellation

A centroidal Voronoi tessellation is a more restrictive definition of Voronoi tessellation.

The constrain for the centroidal Voronoi tessellation is simply that each Voronoi generator must be the mass centroid for its corresponding Voronoi region.

For example in the figure it may not be apparent at first glance but Voronoi graph has its generators in the exact place of the mass centroids of the Voronoi regions. Thus it is a Centroidal Voronoi diagram.¹¹

In Diagram 10, for a simple two generator example of this non-uniqueness.

Each has the same density function, domain, and each has two generators.

Both diagrams in Figure 3 and 4 satisfy the conditions of a CVT. Honeycomb geometry is a CVT.

Examples of Centroidal Voronoi tessellation

Diagram 11

Above – Set of points arranged in a regular square grid.

Below – Set of points act as generator and as centroid for each polygon resulting into a square network.

Diagram 12

Above – Set of points in arranged in a hexagonal grid.

Below – Set of points act as generator and as centroid for each polygon resulting into a hexagonal network.

Diagram 13

Above – Set of points arranged in an order where distance in X axis and Y axis is not the same but they remain constant in their respective direction.

Below – Set of points act as generator and as centroid for each polygon resulting into a network of six sided polygons.

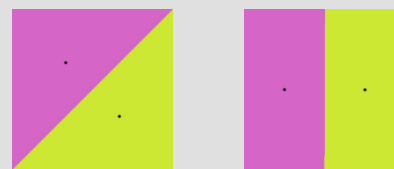


Diagram 10 – Points are the centre of mass of their respective sites.

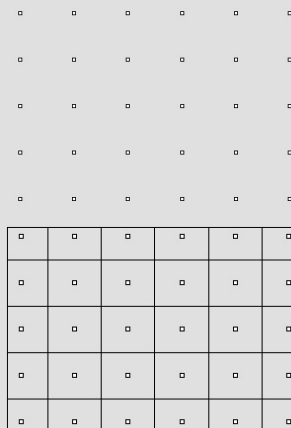


Diagram 11 – Above. Arrangement or points in a square grid.

Below – Generated Centroidal Voronoi diagram from these points.

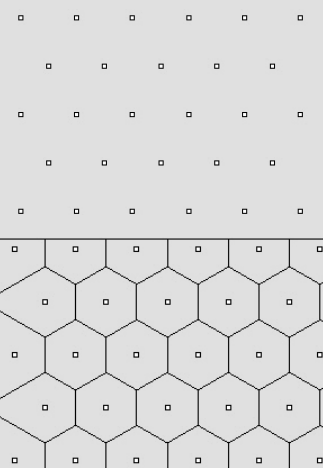


Diagram 12 – Above . Arrangement or points in a hexagonal grid.

Below – Generated Centroidal Voronoi diagram from these points.

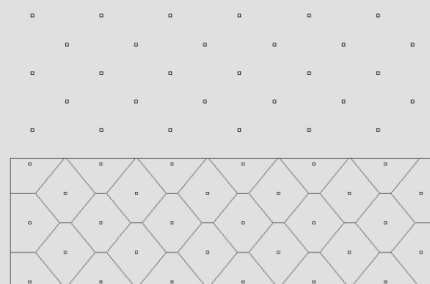


Diagram 13 – Above . Arrangement or points in a grid.

Below – Generated Centroidal Voronoi diagram from these points.

¹¹ Centroidal Voronoi Tessellation – Jared Burns

Diagram 14

Left: 10 randomly selected points in the square (the filled circles), the associated Voronoi cells (the polygons), and the centre of mass of the Voronoi cells with respect to a constant density (the open circles); note that the generating points and centroids do not coincide.

Right: A 10 point Centroidal Voronoi tessellation (CVT) of the square; note that open circles are both the generating points of the Voronoi tessellation and the centroids of the Voronoi cells.

Diagram 15

Above: the Voronoi tessellation of 256 uniformly distributed randomly selected points in the square.

Below: a 256-point CVT of the square corresponding to a uniform density.

Diagram 16

Above: the Voronoi tessellation of 256 nonuniformly distributed randomly selected points in the square.

Below: a 256-point CVT of the square corresponding to a nonuniform density.¹²

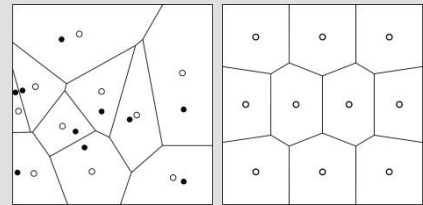


Diagram 14 – Left. A Voronoi diagram
Right. A Centroidal
Voronoi diagram.

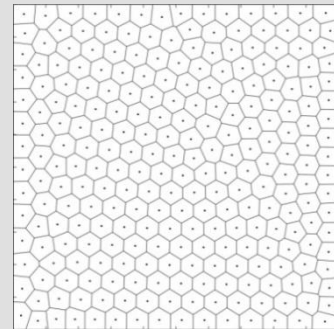
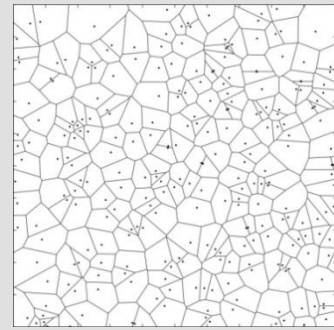


Diagram 15 – Above. A Voronoi
diagram
Below. A Centroidal
Voronoi diagram.

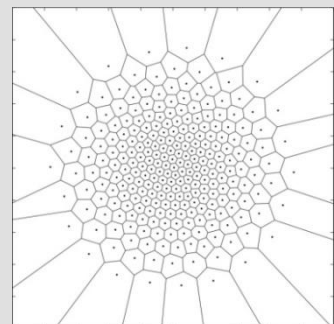
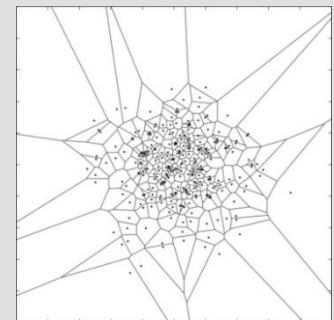


Diagram 16 – Above. A Voronoi
diagram
Below. A Centroidal
Voronoi diagram.

¹² <http://people.sc.fsu.edu/~mgunzburger/cvt/cvt1.html>

Lloyd's algorithm –

In computer science and electrical engineering, Lloyd's algorithm, also known as Voronoi iteration or relaxation, is an algorithm for grouping data points into a given number of categories, used for k -means clustering.

k -means clustering is a method of cluster analysis which aims to partition n observations into k clusters in which each observation belongs to the cluster with the nearest mean

Lloyd's algorithm is usually used in a Euclidean space, so the distance function serves as a measure of similarity between points, and averaging of each dimension for the averaging, but this need not be the case.

Lloyd's algorithm starts by partitioning the input points into k initial sets, either at random or using some heuristic (experience-based techniques for problem solving, learning, and discovery. Heuristic methods are used to speed up the process of finding a good enough solution, where an exhaustive search is impractical. Examples of this method include using a "rule of thumb", an educated guess, an intuitive judgment, or common sense.). It then calculates the average point, or centroid, of each set via some metric (usually averaging dimensions in Euclidean space). It constructs a new partition by associating each point with the closest centroid, usually using the Euclidean distance function. Then the centroids are recalculated for the new clusters, and algorithm repeated by alternate application of these two steps until convergence, which is obtained when the points no longer switch clusters (or alternatively centroids are no longer changed).

More formally:

Lloyd's algorithm starts with an initial distribution of samples or points and consists of repeatedly executing one relaxation step:

- The Voronoi diagram of all the points is computed.
- Each cell of the Voronoi diagram is integrated and the centroid is computed.
- Each point is then moved to the centroid of its voronoi cell.

Each time a relaxation step is performed, the points are left in a slightly more even distribution: closely spaced points move further apart, and widely spaced points move closer together. In one dimension, this algorithm has been shown to converge to a Centroidal Voronoi diagram, also named a Centroidal Voronoi tessellation (Du, Emelianenko & Ju 2006). In higher dimensions, some weaker convergence results are known (Sabin 1986).

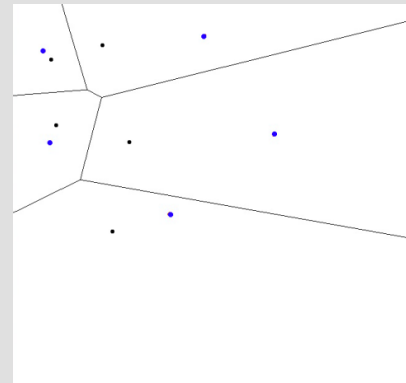


Diagram 17 a - step 1 creating Voronoi diagram

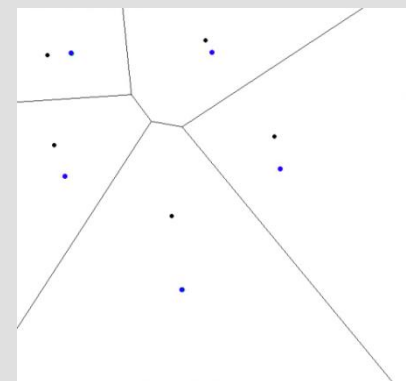


Diagram 17 b – step 2 shifting position to form centre of mass

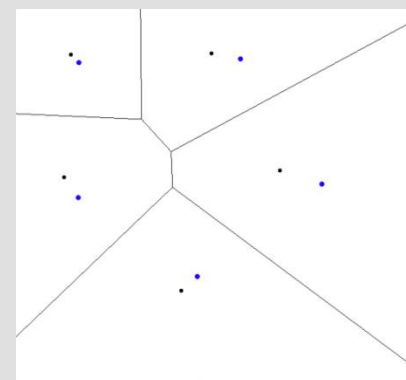


Diagram 17 c – step 3

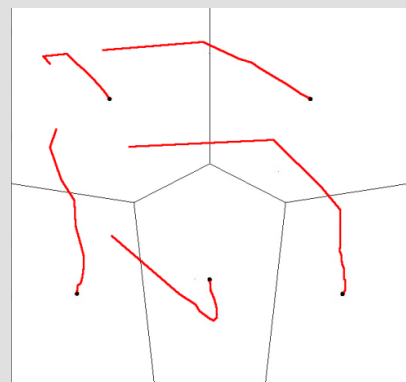


Diagram 17 d – placing at centre of mass, red line show trajectory path

When and Why Centroidal Voronoi?

When comparison is drawn between 2D Voronoi and Centroidal Voronoi, the only difference is that Centroidal Voronoi is a bit restrictive. Centroidal Voronoi has a much defined parameters of its own that the generator points has to be the centre of mass of the respective resultant domains.

Thus its application in a design process is almost same as of 2D Voronoi. But Centroidal Voronoi can generate outputs which are visually more stable.

The inherent parameter of controlling the relation between generator points and related domain gives an inbuilt structure of it's own to this geometry. This inherent structural quality of its own binds itself to any design variable and thus create a responding resultant.

3D Voronoi.

The definition of Voronoi diagram in the plane also works in three-dimensions.

In any dimensions, the VD has a geometric dual structure called the Delaunay Triangulation. In 2D, this structure is defined by the partitioning of the plane into triangles.

The generalisation to three dimensions of the Delaunay triangulation is the **Delaunay tetrahedralization**: each triangle becomes a tetrahedron that satisfies the empty circumsphere rule.

The DT is unique for a set of points, except when there are degenerate cases in the set (if five or more points are cospherical in 3D). In these cases, an arbitrary choice must be made among all the possible solutions. The number of tetrahedra in a DT constructed with n points depends on the configuration of these points, and can be up to $O(n^2)$.

Most of the properties of the 2D VD/DT generalise to 3D, except that the minimum angle in each Delaunay tetrahedra is not maximized. There can indeed be almost "flat" Delaunay tetrahedra. These tetrahedra, called slivers, have their four vertices almost lying on a plane and thus have a volume of nearly zero.

For many applications where the Delaunay tetrahedralization is used, e.g. to perform simulation in engineering or when the tetrahedra are used to perform interpolation directly, these tetrahedra are bad and must be removed.

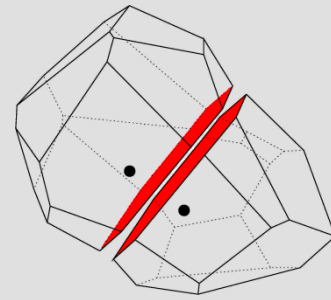


Diagram 18 -Two adjacent Voronoi cells. The grey face is the face they share. –

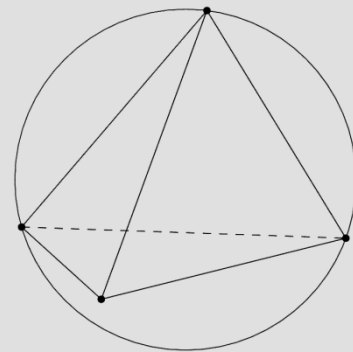


Diagram 19 -A Delaunay tetrahedron has an empty circumsphere

The input points or the generator of Voronoi cells are arbitrarily located in space.

The Voronoi cells are convex polyhedrons.

The faces bisect planes of input points. Edges are intersections of (in general) three bisecting planes (axis of a circum-circle of three input points), and at the vertices four (or more) bisecting planes meet. Of course, the vertices are centres of spheres through four (in special cases, more) input points¹³

Point set construction of three points in a cube.

- The defined cube acts as a bounding box.
- Fig 20.a
3 random points located in a cube.
- Fig 20.b
Construction of Delaunay triangulation over these points.
- Fig20.c
Generating planes as perpendicular bisectors of the line segments

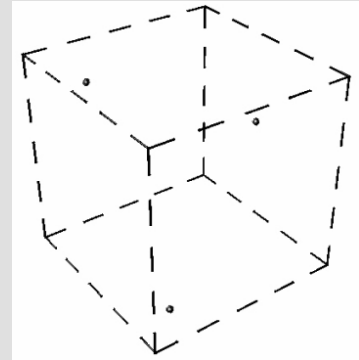


Diagram 20.a – Input points inside a cube.

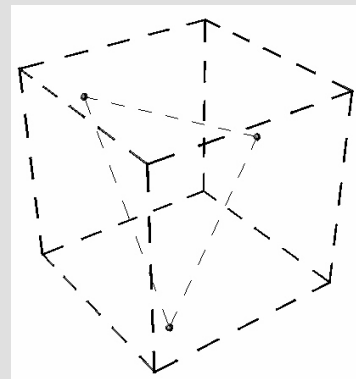


Diagram 20.b – Delaunay triangulation.

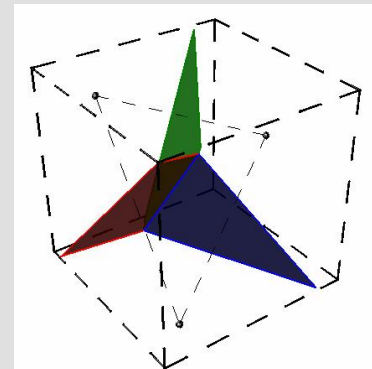


Diagram 20.c – 3d Voronoi diagram..

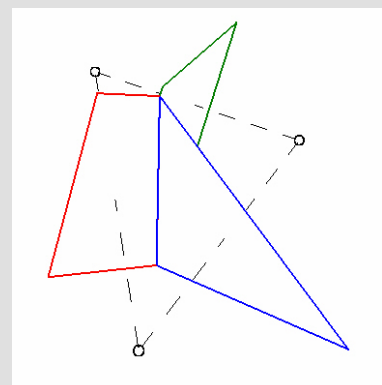


Diagram 20.d – 3d Voronoi diagram..

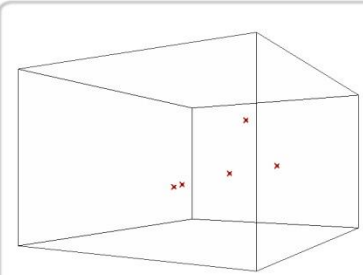
¹³ Architectural Geometry.

Helmut Pottmann, Andreas Asperl, Micheal Hofer, Axel Kilian

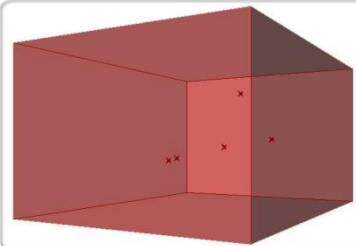
Chart 2 -Creation of 3d Voronoi through random set of points.



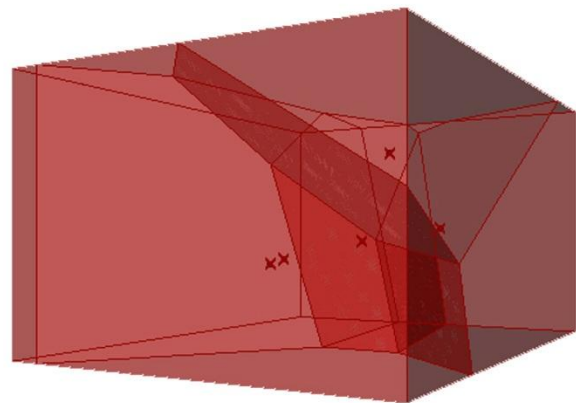
Plan showing the arrangement of points.



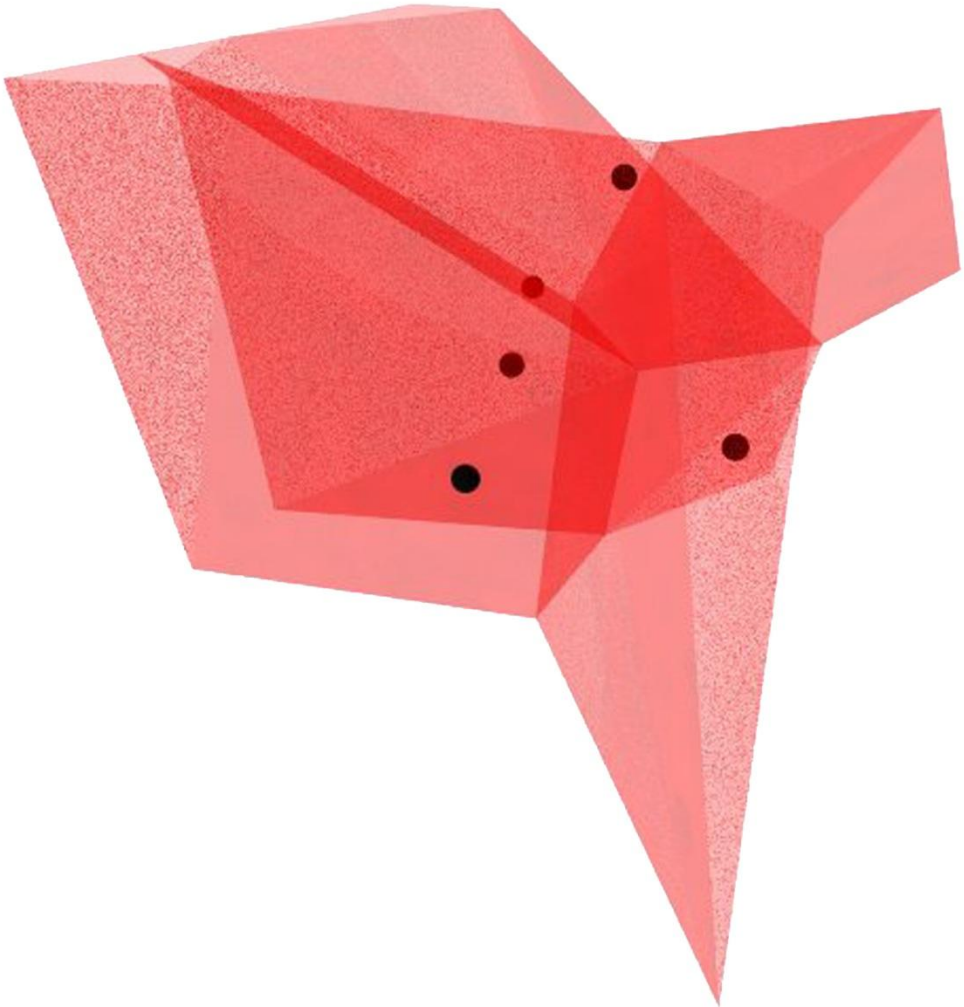
Generating five random points in space



Defining a bounding box, which defines the limit for the generation of voronoi



The planes generated inside are the voronoi planes, not the given points are the input data, and the bounding box is also an input, which is called Brep in this definition

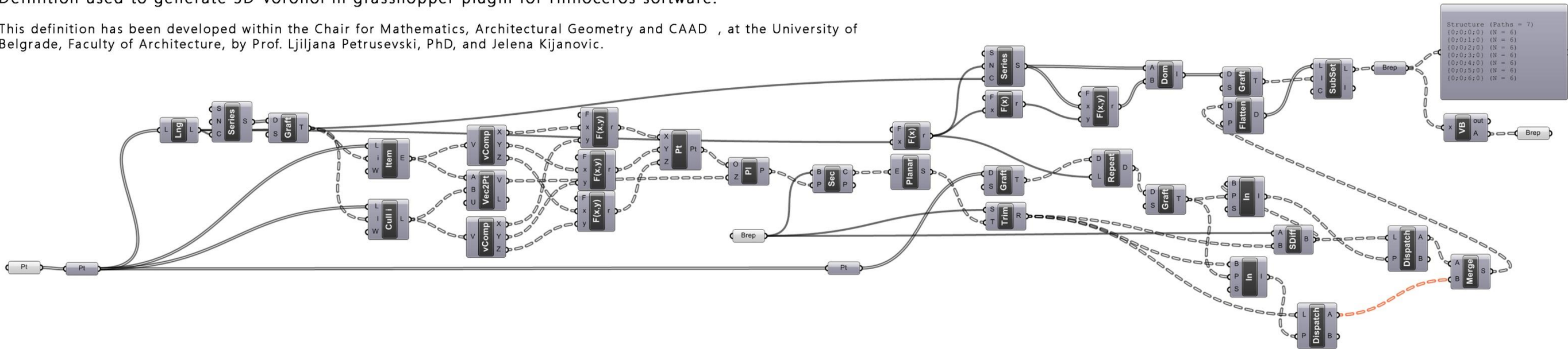


Final Output
3D Voronoi of the random set of points.
The black spheres are the points(highlighted)

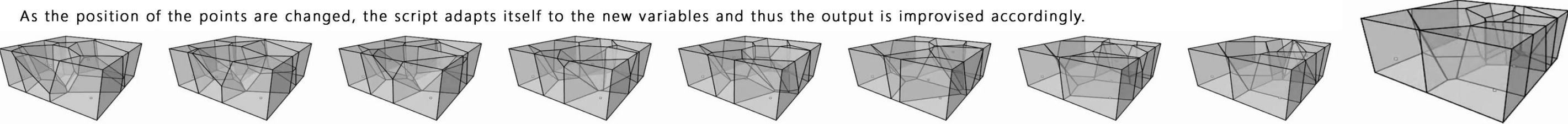
The output is confined with respect to the bounding box defined at the earlier stage

Definition used to generate 3D Voronoi in grasshopper plugin for rhinoceros software.

This definition has been developed within the Chair for Mathematics, Architectural Geometry and CAAD , at the University of Belgrade, Faculty of Architecture, by Prof. Ljiljana Petrusevski, PhD, and Jelena Kijanovic.



As the position of the points are changed, the script adapts itself to the new variables and thus the output is improvised accordingly.



When and Why 3D Voronoi?

Understanding application of 3D Voronoi in design depends on a lot more factors than just this simple categorization of programme, concept and context. (This is understood in detail later on along with case studies.)

Design Programme and 3D Voronoi:

Application of 3D Voronoi can be seen similarly as 2D Voronoi. But in 3D Voronoi design programme can be translated as a system model. Vaguely, it can be described as a three dimensional flow chart. It can be applied for a definitive as well as a generative programmatic approach.

Concept and 3D Voronoi:

When applying such an approach with a conceptual strength, it should be understood as not just form making tool. It can be looked upon as a very productive and proactive system, with open ends.

Context and 3D Voronoi:

Contextual responses on 3D Voronoi are the same as 2D Voronoi.

Other types of

Voronoi diagrams

Nearest-Neighbour
Voronoi Diagrams in the plane

Diagram 21.a - A standard Voronoi diagram (point sites with Euclidean metric).

Diagram 21.b - An additively-weighted Voronoi (Apollonius) diagram with disk centres as sites and disk radii as weights.

Diagram 21.c - A Mobius diagram with disk centres as sites. The distance from every point on the boundary of a disk to its corresponding site is zero.

Diagram 21.d - A Voronoi diagram of segments.

*The sites in (21.b), (21.c), and (21.d) are displayed as dashed curves.

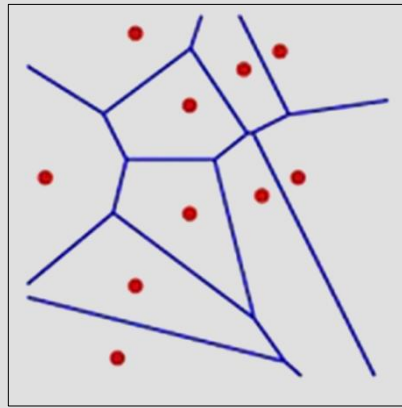


Diagram 21.a.

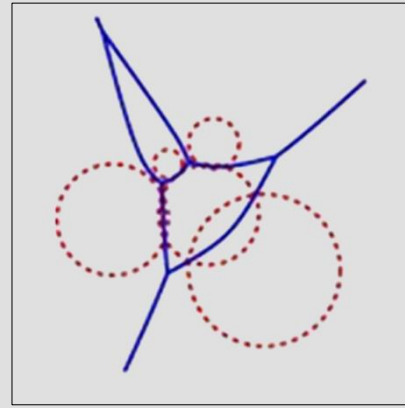


Diagram 21.b.

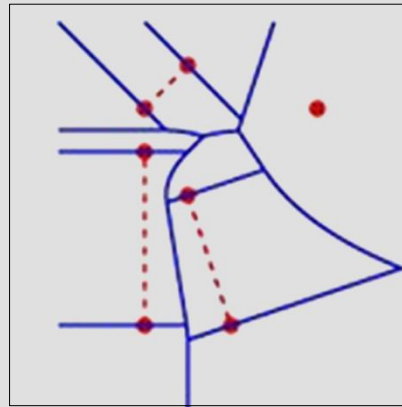


Diagram 21.c.

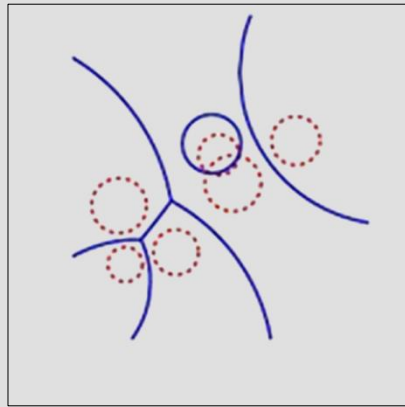


Diagram 21.d

Farthest- Neighbour Voronoi
Diagrams in the plane

(a) A farthest point Voronoi diagram.

(b) A farthest additively-weighted Voronoi diagram.

(c) A farthest Mobius diagram.

(d) A farthest Voronoi diagram of segments.

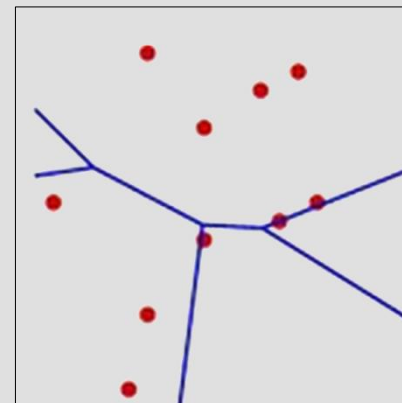


Diagram 22.a.

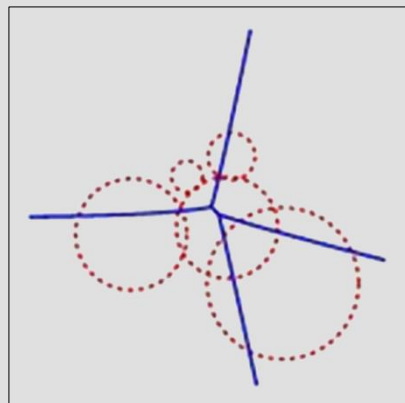


Diagram 22.b.

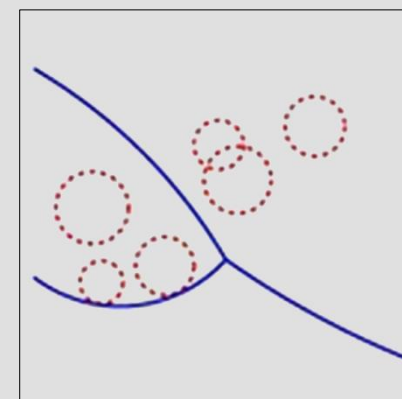


Diagram 22.c.

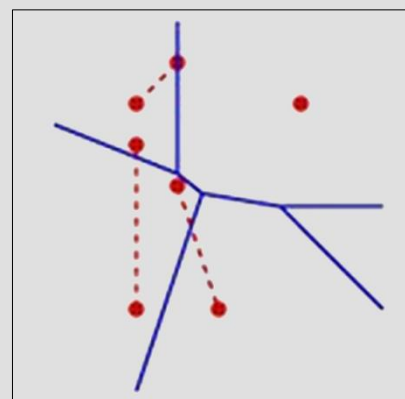


Diagram 22.d

Degenerate Voronoi
Diagrams in the plane.

(a) A degenerate standard
Voronoi diagram.

(b) A degenerate additively-
weighted Voronoi
(Apollonius) diagram.

(c) A degenerate Voronoi
diagram of segments¹⁴

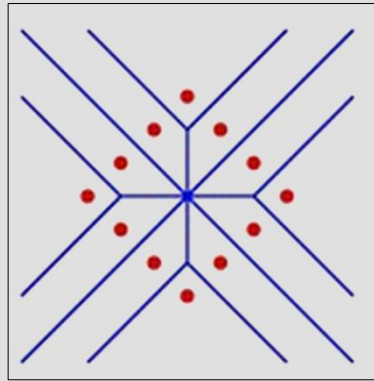


Diagram 23.a.

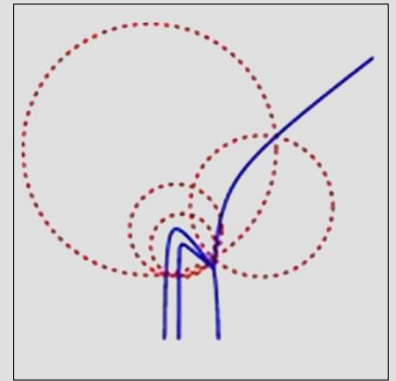


Diagram 23.b.

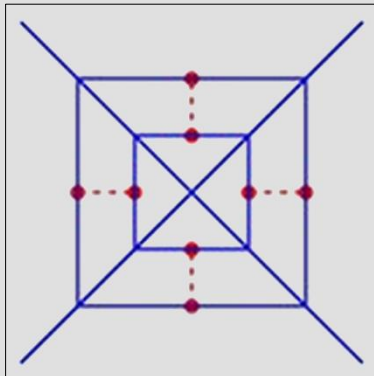


Diagram 23.c.

Voronoi Diagrams on the
sphere

(a) The Voronoi diagram of 32
random points.

(b) A highly degenerate case
of Voronoi diagram of 30
point sites on the sphere.

(c) The power diagram of 10
random circles.

(d) A degenerate power
diagram of 14 site on the
sphere.

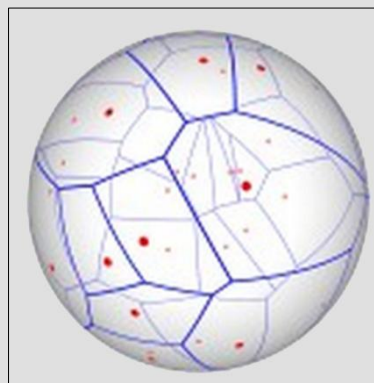


Diagram 24 a.

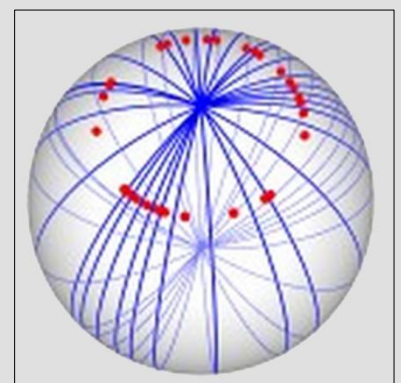


Diagram 24 b.

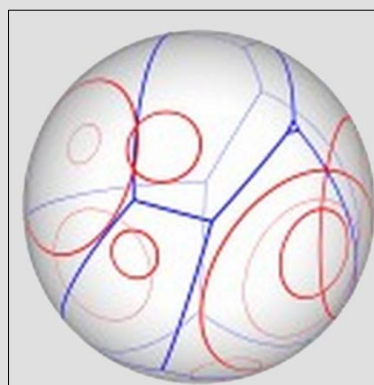


Diagram 24.c.

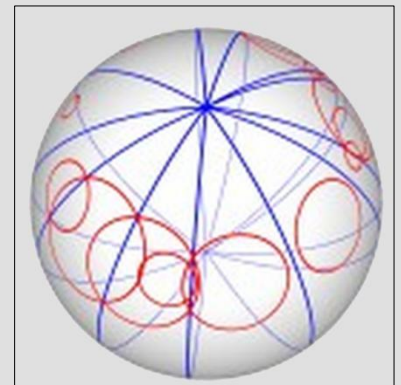


Diagram 24.d.

¹⁴ <http://acg.cs.tau.ac.il/projects/internal-projects/vd-via-dc-of-envelopes>

Voronoi diagrams are among very important structures in computational geometry. A Voronoi diagram records information about what is close to what. Some of the most important operations in computational morphology is that of “growing” and “shrinking” (or “thinning”) objects. The medial axis of a shape (used in computer vision) is just a Voronoi diagram of its boundary¹⁵

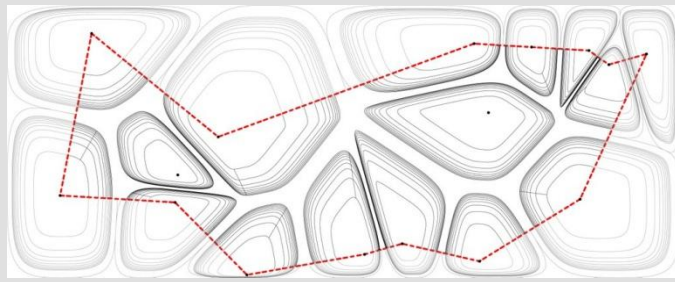


Figure 3. A computer generated image of Voronoi.

¹⁵ www.cs.wustl.edu/~pless/546/lectures/l16.html



In recent years, new strategies for design and new techniques for making materials and large constructions have emerged, based on biological models of the processes by which natural material forms are produced.

Biological organisms have evolved multiple variations of form that should not be thought of as separate from their structure and materials. Such a distinction is artificial, in view of the complex hierarchies within natural structures and the emergent properties of assemblies. Form, structure and material act upon each other, and this behavior of all three cannot be predicted by analysis of any one of them separately.

Natural materials develop under load, and the intricate interior structure of biological materials is an evolutionary response.¹⁶

Voronoi geometry in nature

Voronoi diagrams tend to be involved in situations where a space should be partitioned into "spheres of influence", including models of crystal and cell growth as well as protein molecule volume analysis. This principle seems to govern many processes of growth and distribution in nature.¹⁷

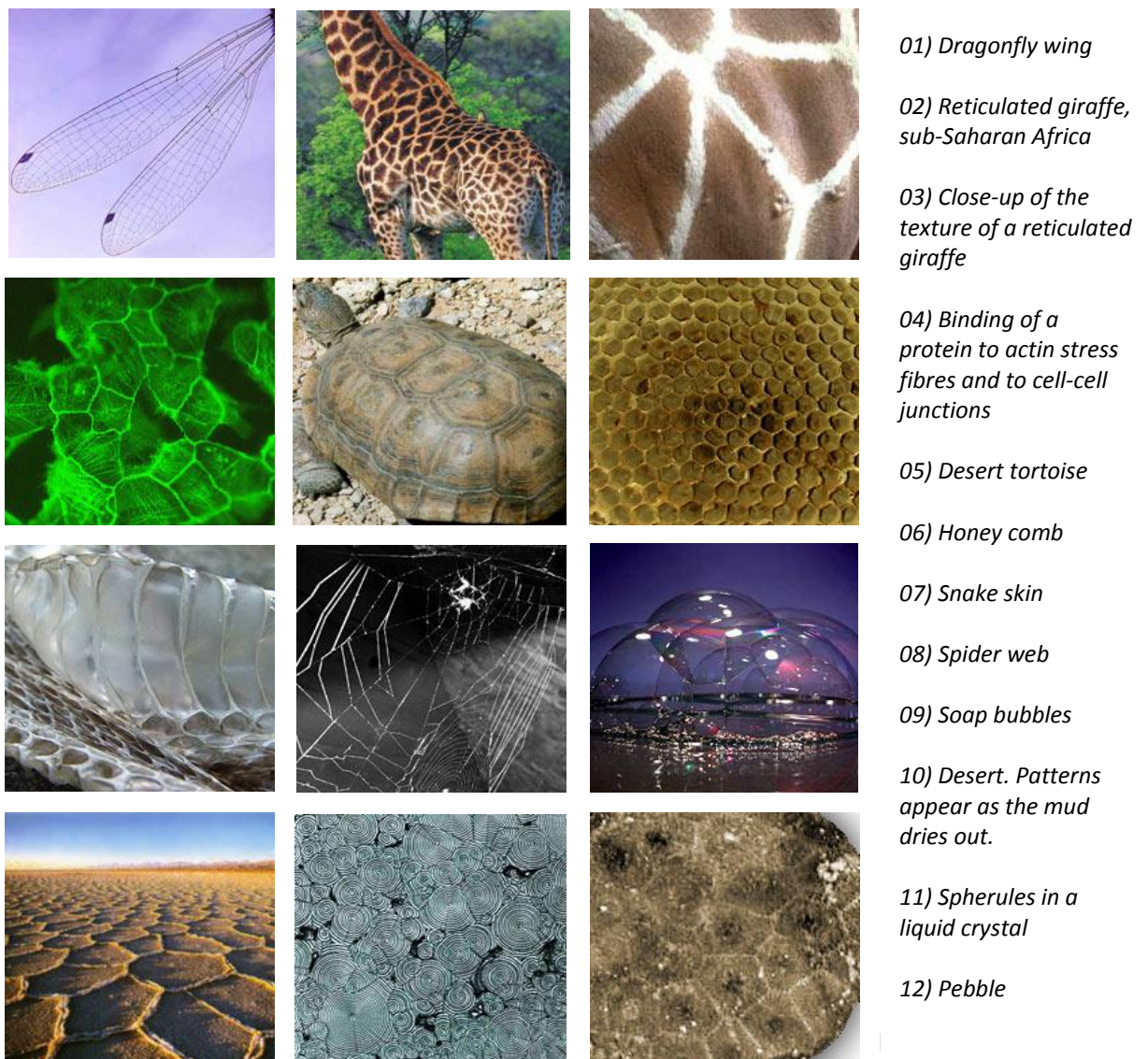


Figure 4 – Examples from nature showing presence of Voronoi Geometry.

¹⁶ Self-Organization and material construction -- Weinstock

¹⁷ D'Arcy Wentworth Thompson, On Growth and Form 1917
V.L Hansen, Geometry in Nature, A K Peters, Ltd, Wellesley, Massachusetts, US

Dragonflies fulfil amazing flight manoeuvres. They are able to remain stationary in one position in the air and to change to any flight direction instantaneously. These remarkable aerodynamic properties are caused by two main components: the musculature and the wing geometry.

Dragonfly wings in nature are generated by evolutionary processes involving aerodynamics, lightness, mechanical properties, composite performance, the smooth accumulation of organic material, and the active flow of dragonfly blood.

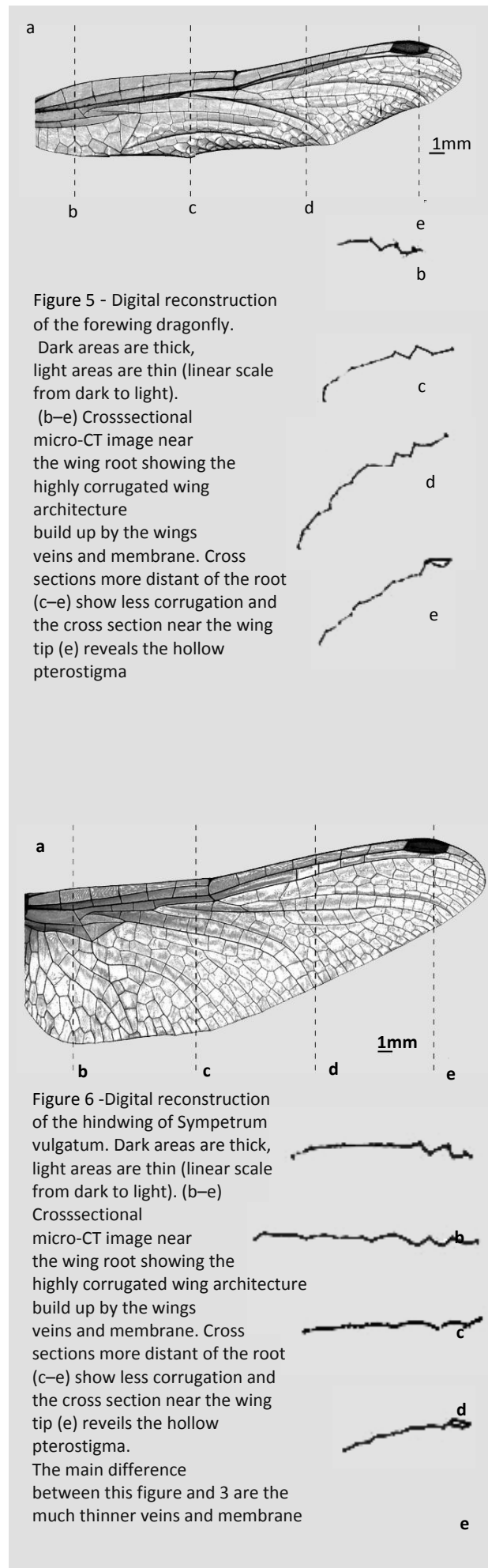
One wing weighs only about 3% of the whole dragonfly [Kesel et al, 1993], but at the same time is very robust and stiff.

During gliding, dragonfly wings can be interpreted as acting as ultra-light aerofoils which, for static reasons, have a well-defined cross-sectional corrugation (see figure 1). This corrugation forms profile valleys in which rotating vortices develop [Hien et al., 1996]. The cross-sectional configuration varies greatly along the longitudinal axis of the wing.

“the morphology of the dragonfly wing is an optimal natural construction via a complex patterning process, developed through evolution as a response to force flows and material organization. The wing achieves efficient structural performance through a nonlinear variation of pattern, corrugations and varied material properties throughout the structure.”

Maria Mingallon Ma'am

i



Analysis of Dragonfly wing in terms of patterns

“The wing is traversed by a few strong “veins” or ribs, more or less parallel to one another, between which finer veins make a meshwork of “cells”, these lesser veins being all much of a muchness exerting tensions insignificant compared with those of greater veins.”¹⁸

The random variation in the natural pattern of the wings were in fact optimized to allow rigid and flexible configurations along the span of the wings that allow for a logic based use of ambient energy for the purposes of flight.

Based on the diagram the general geometrical conclusions arrived at by the team were as follows:

1. The patterns of the wings follow the general tensile forces exhibit on the wing
2. The various shapes carry the responsibility of determining the amount of stiffness or flexibility in that area of the wing.

For example the quadrilateral areas on the edges determine the more rigid and stiff portions of the wing while the largely compartmentalized hexagonal areas are responsible for the areas more likely to bend and sway. Furthermore, connections between the cells also determined the degree to which adjacent cells were free to bend, that was also highlighted in the research:

“Two main types of joints occur in the dragonfly wings, mobile and immobile. Some longitudinal veins are elastically joined with cross veins, whereas other longitudinal veins are firmly joined with cross veins. Scanning electron microscopy reveals a range of flexible cross-vein and main-vein junctions in the wing, which allows local deformations to occur. The occurrence of

resilin, a rubber-like protein, in mobile joints enables the automatic twisting mechanism of the leading edge.”¹⁹ The arrangement of cells in dragonfly wing is orderly and simple. The long narrow wing is stiffened by longitudinal “veins”, which in front lie near and parallel for reasons of aerodynamics, but become divergent and remote over the rest of the wing; finer veinlets running between the veins, break up the surface into cells of areolae.

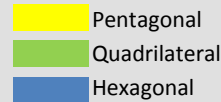
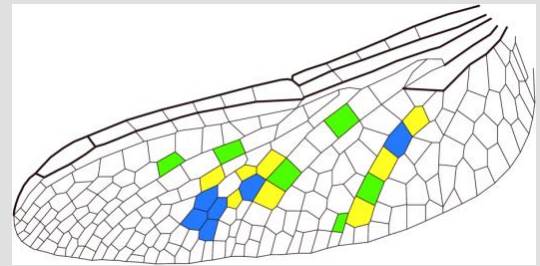


Figure 7 -A diagram to showing the dragonfly wing which is divided into various shapes that are designed to handle the forces differently.

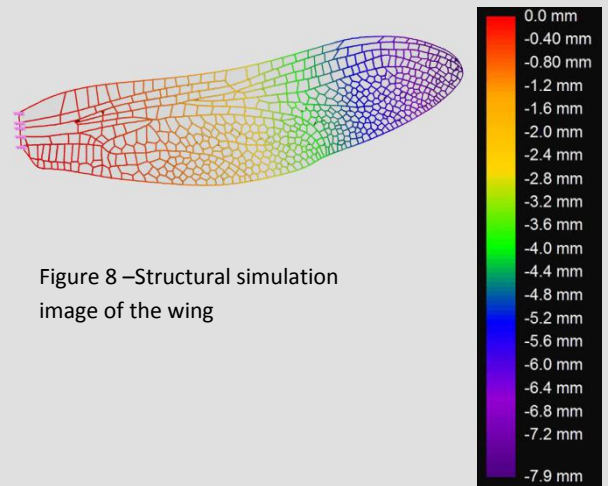


Figure 8 –Structural simulation image of the wing

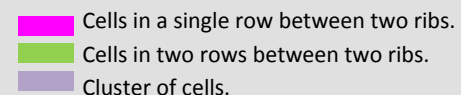
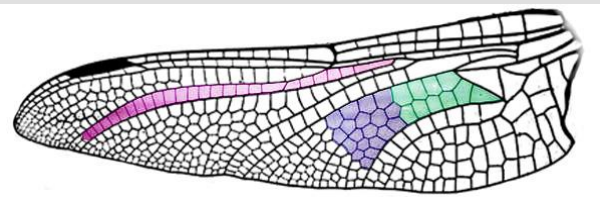


Fig 9 – Image showing different types of cells .

¹⁸ Structure in nature is a strategy for Design. Peter Pearce.

¹⁹ Maria Mingallon and her team's research

In dragonfly wing three different types of patterns are seen.

All these patterns tend to achieve minimum length based on their related parameters.

- Single row of cells meeting two ribs.

Where two ribs run so near together that only one row of cells lie between, these cells are quadrangular in form, their thin partitions meeting the ribs at right angles on either side.

The shortest length to connect two lines is by drawing a line perpendicular to any one of them. This same logic is applied in this situation when the distance between two ribs is very less and no more connectors are required in between them.

- Two rows of cells meeting each other as well as the ribs.
11.61 mm for 120 degree angle.
11.75 mm for voronoi.

Where (diagram 25.b), two rows of cells are intercalated between a pair of ribs, one row fits into the other by angles of 120 degrees, the result of co-equal tensions; but both meet the ribs at right angles, as in the former case and the cells resolve, consequently, into a hexagonal network.²⁰

The resultant connection between the rows is at an angle of 120 degrees because the rows meet the ribs at 90 degrees on each side.

Voronoi diagram based on random set of points does not appear an optimised method in such situation as there is a defined parameter of each row meeting the ribs at 90 degrees (diagram 25.c)

- Cluster of cells.

When the cell-rows are numerous, all their angles in common tend to be co-equal angles of 120 degrees, and the cells resolve, consequently, into a hexagonal mesh-work.

It consists of Centroidal Voronoi tessellation.

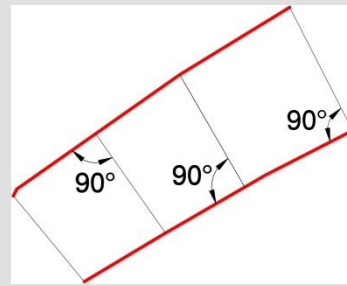


Diagram 25.a – single row of cells meeting two ribs.

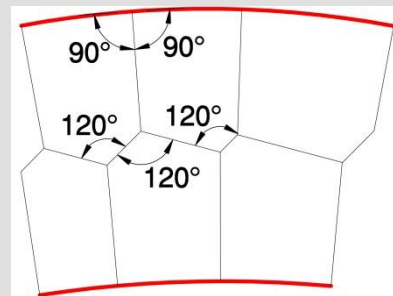


Diagram 25.b – Two rows of cells meeting each other as well as the ribs

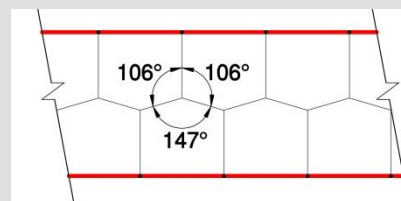


Diagram 25.c – Voronoi diagram exhibiting angles not at 120 degrees.

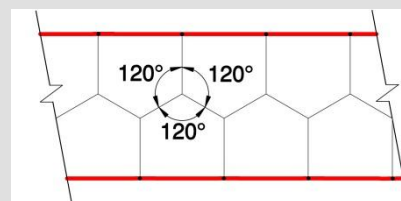


Diagram 25.d – The resultant connection between the rows.

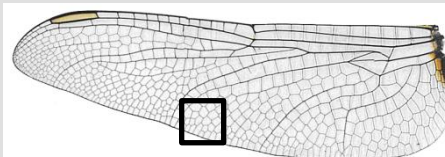
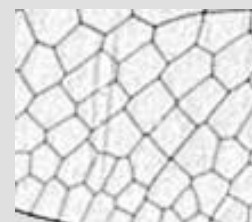


Diagram 25.e – Centroidal Voronoi Geometry formed by a cluster of cells.



²⁰ Structure in Nature is a Strategy for Design – Peter Pearce.

This biomimetic based research study investigates how optimal natural construction results from complex patterning processes as a response to loading paths and material organisation. Highly efficient structural performance is achieved through a nonlinear variation of pattern geometries, corrugations and varied material properties. The wing geometry transforms into different configurations to utilise the ambient wind flow energy for their flight.

Soap Bubbles

The Belgian physicist Joseph A.F. Plateau (1801 – 1883) made soap bubbles the subject of experimental observations and scientific study. He immersed wire loops of various shapes in a soap solution and was able to find a minimal surface for each given boundary because the soap film always form a minimal surface due to its surface tension. A soap film regularly contracts to the smallest surface possible. It then takes up the form of the minimal surface, which is clearly defined mathematically.

Soap films seek to minimise their surface area, that is, to minimise their surface energy. The optimum shape for an isolated bubble is thus a sphere. This is a state of minimum potential energy. When a soap bubble comes in contact with other cells, it becomes more economical for the sphere to change into a polyhedron.

Figure 10 – Intersection of two soap bubbles of the same size²¹

Figure 11 – Intersection of three soap bubbles of same sizes.

Figure 12 – Intersection of four soap bubbles of same sizes

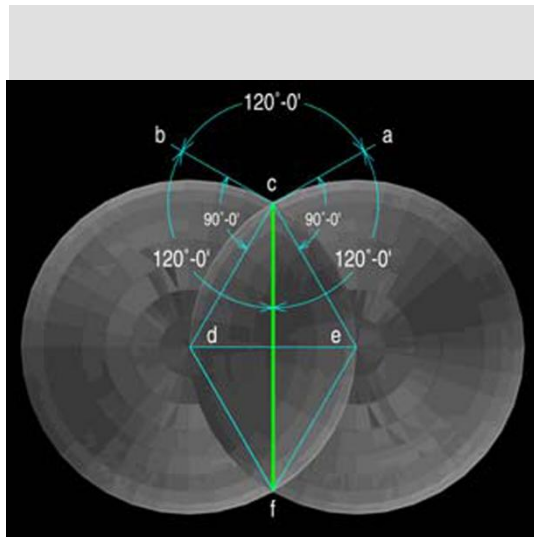


Figure 10 – Intersection of two soap bubbles of same size

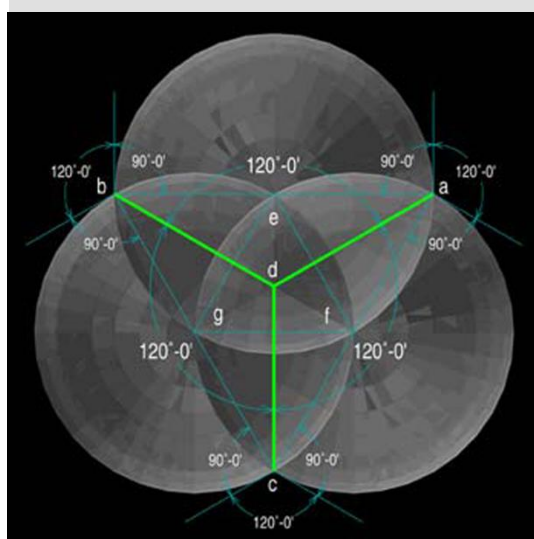


Figure 11 – Intersection of three soap bubbles of same size

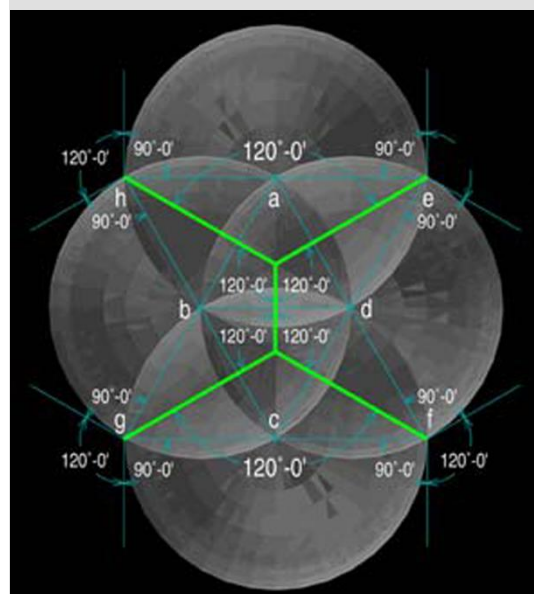


Figure 12 – Intersection of four soap bubbles of same sizes

²¹ GROWTH, FORM AND PROPORTION IN NATURE: LESSONS FOR HUMAN HABITATION OFF PLANET ENVIRONMENTS – Vancouver paper, Micheal Kreigh . Architect Kalil Endowment for Smart Design.

When we have three bubbles in contact, they are separated by three partition surfaces, whose curvature will depend upon the relative size of the spheres, and which will be plane if the latter are all of equal size; but whether plane or curved, the three partitions will meet one another at angles of 120 degrees, in an axial line. Various pretty geometrical corollaries accompany this arrangement, if (diagram 26.c) represent the three associated bubbles in a plane drawn through the centres, C , C' , C'' (or what is the same thing, if it represent the base of three bubbles resting on a plane), then the lines UC , UC' , or SC , SC' , etc., drawn to the centres from the points of intersection of the circular arcs, will always enclose an angle of 60 degrees. Again diagram (26.c) if we make the angle $C''UF$ equal to 60° , and proceed UF to meet C'' in F , F will be the centre of circular arc which constitutes the partition OU ; and further, the three points F , G , H , successively determined in this manner, will lie one and the same straight line.

In the case of three co-equal bubbles (as in diagram 26.a), it is obvious that the three lines joining their centres form an equilateral triangle: and consequently, that the centre of each circle (or sphere) lies on the circumference of the other two; it is also obvious that UF is now parallel to CC'' , and accordingly that the centre of curvature of the partition is now infinitely distant, or (as we have already said) that the partition itself is a plane.²²

- In every liquid system of thin films in stable equilibrium, the sum of the areas of the films is a minimum;
- the area of each is a minimum under its own limiting conditions;
- the mean curvature of any film is constant throughout its whole area, null when the pressures are equal on either side and in other cases
- proportional to their difference;
- the films meeting in any one edge are three in number;
- the crests or edges meeting in any one corner are four in number, neither more nor less;
- the three films meeting in a crest or edge do so at co-equal angles, and the same is true of the four edges meeting in a corner.

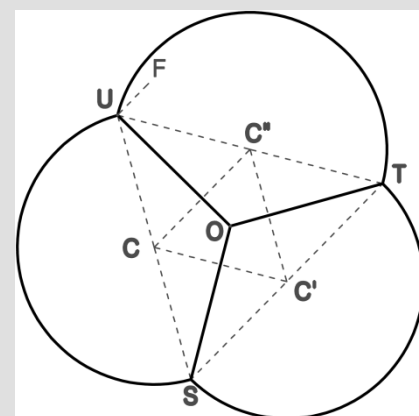


Diagram 26 a – Intersection of three soap bubbles of same size

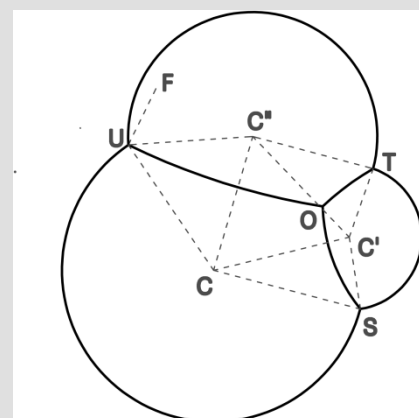


Diagram 26.b – Intersection of three soap bubbles of different sizes

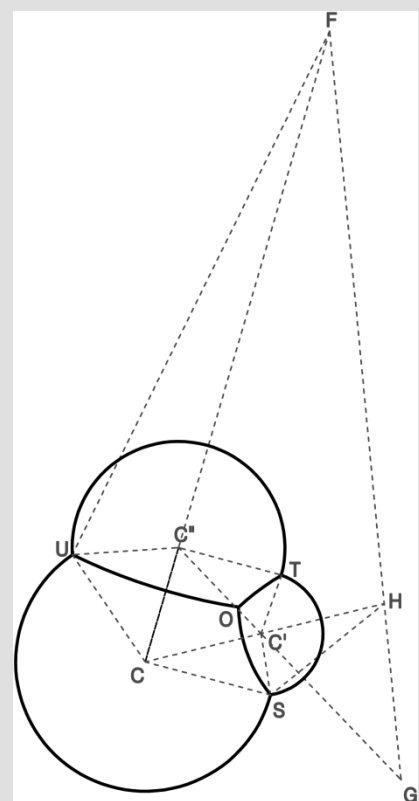


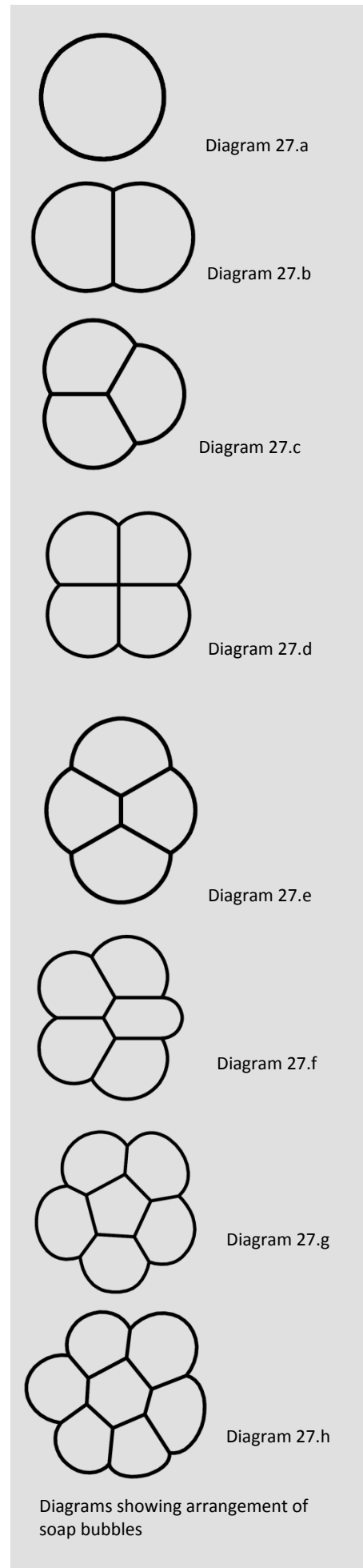
Diagram 26.c – Intersection of three bubbles of different sizes

²² On Growth and Form – Volume 2.
D' Arcy Thompson.

When there are four bubbles meeting in a plane (diagram 27.d and diagram 27.e), they would seem capable of arrangement in two symmetrical ways: either (diagram 27.d) with four partition-walls intersecting at right angles, or (diagram 27.e) with five partition –walls, three and three, at angles of 120° .

The diagram 27.d arrangement is strictly analogous to the arrangement of three bubbles in diagram 27.c. Now, though both of these figures might seem, from their apparent symmetry, to be figures of equilibrium, yet in point of fact the latter turns out to be of stable and the former of unstable equilibrium. If we try to bring four bubbles into a form (diagram -27.d), that arrangement endures only for an instant; the partitions glide upon one another, an intermediate wall springs into existence, and the system assumes the form (diagram – 27.e), with its two triple, instead of one quadruple conjunction.

In three-dimensional array of bubbles the rule appears to change to the extent that there are always four edges meeting at each vertex at angles of $109^\circ 28'$. In actuality, the view of three edges meeting at a point at 120° is simply a plan (cross sectional) view of the three partition faces meeting on a common edge at 120° angles.²³



²³ Structure in Nature is strategy for design. Peter Pearce

Relationship between soap bubbles and Voronoi Geometry

If in some partition system, perhaps polyhedral, the partition system is let to relax until the cost is a minimum, at least among nearby partitions, then the resulting stable partition should exhibit the geometry of a cluster of soap bubbles. Thus a stable partition should follow the rules recorded by Plateau [Pla] in 1873 for such clusters, and proved by Jean Taylor [Tay] in 1976 for a certain mathematical model (due to Fred Almgren) of compound bubbles. (Sullivan)

These Plateau rules state that the interfaces are smooth surfaces of constant mean curvature, except where they meet in threes (at equal 120° angles) along smooth arcs. These arcs are, furthermore, allowed to come together (four at a time) at isolated points, where the configuration is tetrahedral; but no other singularities are allowed. The allowed singularities are observed for instance in a cluster of three or more soap bubbles, as in Figure ()

The mean curvature of each surface is simply the pressure difference between the two cells on either side; this implies for instance that the three mean curvatures around a triple junction sum to zero. We can view these rules as having a geometric part (about curvatures and equal angles) and a combinatorial part: those cells should meet along edges in threes and at corners in fours, tetrahedrally.

This combinatorial is the same as is observed generically in Voronoi partitions. Given a collection of sites in space, we define the Voronoi cell [Sen, OBS] for each site to be the region consisting of points closer to that site than to any other. Figure () shows an example in the plane. In space, the boundary between two adjacent cells in the partition is a polygonal piece of the plane perpendicularly bisecting the segment between the two sites. These bounding polygons meet along segments equidistant from three sites, which terminate at vertices equidistant from four sites. The Voronoi partition of a symmetric collection of sites will share its symmetry.

This similarity in combinatorial structure suggests we might find foams as relaxed Voronoi partitions. We specify certain sites and compute their Voronoi partition to get polyhedral cells in the right combinatorial structure; then we can follow mean curvature flow (with volume constraints) toward stationary foam satisfying the geometric Plateau rules. This idea was implemented numerically at the Geometry Supercomputer Project in 1988, using Sullivan's vcs software [Sul] to compute Voronoi partitions in three dimensions, and Brakke's evolver to

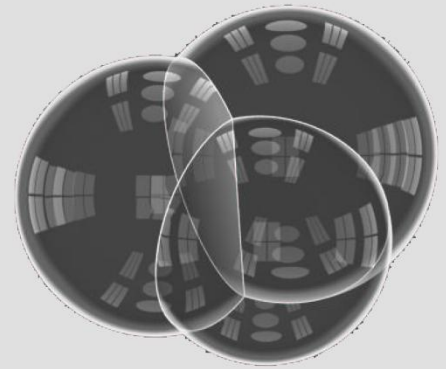


Figure 13 -This triple bubble in space exhibits the Plateau singularities: smooth surfaces meet along triple-junction curves at 120° angles and these junctions come together at tetrahedral points.

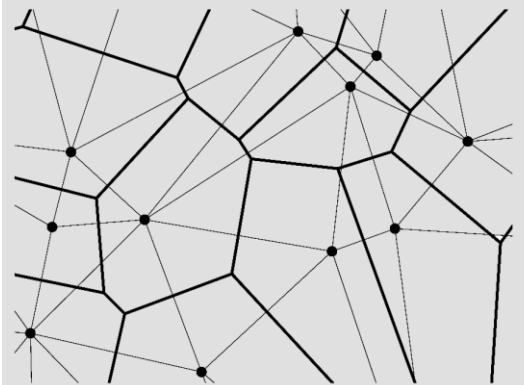


Diagram 28 -Sites in the plane (marked with dots) together with their Voronoi partition (Heavy lines). The thin lines are the dual Delaunay triangulation, connecting sites with adjacent Voronoi cells.

relax them to foams. Some results appeared in the movie \Computing Soap Films and Crystals”, produced by Almgren, Brakke, Sullivan and Taylor, and published in [Alm]. Those experiments found no foams better than Kelvin's. But Weaire and Phelan knew of an interesting pattern (often called A15) arising in chemical catharses, and used the same pair of programs to find that the corresponding foam did have lower cost.²⁴

The cells in a Voronoi partition do not in general have equal volumes. The evolver can adjust the volumes towards the desired targets as the numerical relaxation proceeds. But mathematically it is helpful to start with the correct volumes.

A weighted Voronoi partition is a generalization in which different weights on the various sites change the relative sizes of the cells; each interface plane is moved away from the site of higher weight. With appropriate weights, the initial Voronoi cells will have the desired volumes, and there is less chance that the mean curvature flow will cause combinatorial changes.

Voronoi Figure + Golden Mean – Sun Flower

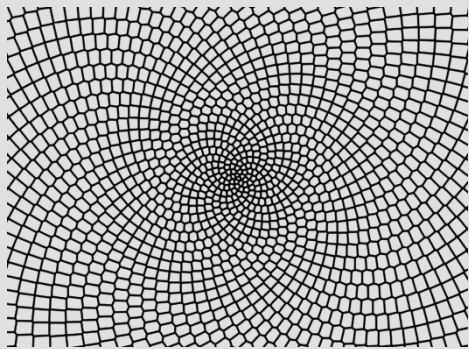


Figure 14 – Internal pattern of sunflower

²⁴ COMPARING THE WEAIRE-PHELAN EQUAL-VOLUME FOAM TO KELVIN'S FOAM
ROB KUSNER AND JOHN M. SULLIVAN

Chapter One. Introduction-Voronoi Geometry	
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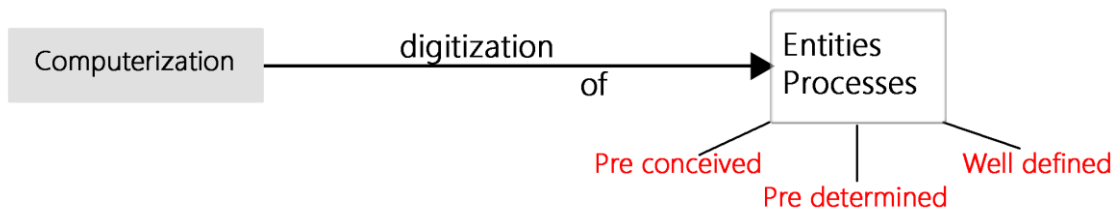
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COMPUTERIZATION.

Computation is a term that differs from, but is often confused with, computerization

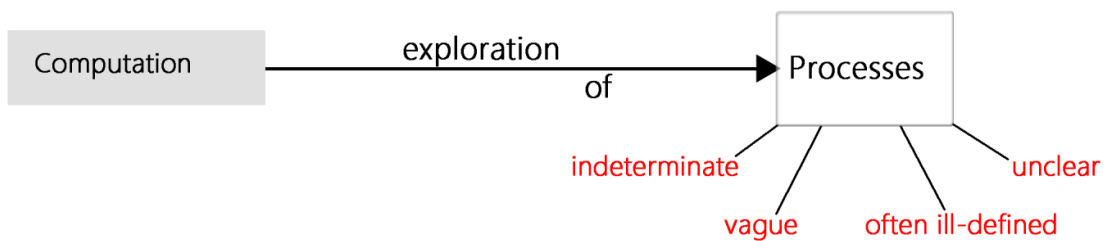
Computerization is about

- Automation
- Mechanization
- Digitization⁴
- Conversion



Computation.

In contrast to computerization, computation is about explorations



Due to its exploratory nature computation aims at emulating or extending the human intellect

Computation is about

- Rationalization
- Reasoning
- Logic
- Algorithm
- Deduction
- Induction
- Extrapolation
- Exploration
- Estimation

In its manifold implication, it involves problem solving, mental structures, cognition, simulation, and rule-based intelligence, to name a few.²⁵

²⁵ Algorithmic Architecture – Kostas Terzidis.

“Algorithms of structure, of travelling lines emerging into shape, open new doors to a radical reappraisal of form generation. Not confined anymore to the strictness of regimented line, non-linear typologies beckon.

But to deny a rigour into investigation of non – linear shape would only promote architectural form as a gesture of software. That could be dangerous, because it is ultimately meaningless. Our brains, random as they are, do not act without interior connections – there are rules – albeit ones hidden to us.

Perhaps intuition has an internal mapping of its own, of non-linear algorithms. Testing this further is a task of the informal”

– Cecil Balmond, Informal.

What is an algorithm?

- In mathematics and computer science an algorithm is an effective method expressed as a finite list of well-defined instructions for calculating a function.
- Algorithms are used for calculation, data processing, and automated reasoning.
- In simple words an algorithm is a step-by-step procedure for calculations.

There are many types of code. They are basically a set of instructions. This type of code is often called an algorithm, procedure, or program. It's just a precise way of explaining how to do something. It is commonly used with the context of computer instructions. While most people wouldn't refer to a pattern for knitting a scarf as an algorithm, it's the same idea²⁶

An algorithm is a process of addressing a problem in a finite number of steps. It is an articulation of either a strategic plan for solving a known problem or a stochastic search towards possible solutions to a partially known problem. In doing so, it serves as a codification of the problem through a series of finite, consistent, and rational steps. While most algorithms are designed with a specific solution in mind to a problem, there are some problems whose solution is unknown, vague, or ill-defined. In the latter case, algorithms become the means for exploring possible paths that may lead to potential solutions.²⁷

An algorithm can be seen as a mediator between the human mind and the computer's processing power. This ability of an algorithm to serve as a translator can be interpreted as bi-directional: either as a means of dictating to the computer how to go about solving the problem, or as a reflection of a human thought into the form of an algorithm.

- A layer of defined set of logistics
- A layer which can arrange design parameters in a specific manner.
- A layer which can help in creating iterations.

Contrary to common belief, algorithms are not always based on a solution conceived entirely in the mind of a human programmer. Many algorithms are simulations of the way natural processes work and as such they must not be regarded as human inventions but rather as human discoveries. .

For Example

```
function "fibonacci", parameters 'n' (integer > 0):  
if n equals 1 return 0;  
if n equals 2 return 1;  
let a = 0; let b = 1;  
while n is greater than 2 do: {  
  let c = a + b;  
  let a = b; let b = c;  
  subtract 1 from n;  
} (end of loop)  
return b;  
(end of function)28
```

An Algorithm for the application of Fibonacci series

Unlike inventions, discoveries are not conceived, owned or controlled by the human mind, yet as abstract processes they can be captured, codified and executed by a computer system. In this case, the human programmer serves the purpose of translating a process external to human mind to be compiled into machine language which is also external to the human mind.

²⁶ Form + Code , Casey Ress, Chandler McWilliams, Lust

²⁷ Algorithmic Architecture.

²⁸ <http://davmac.org/davpage/algrthm/fibonacci.html>

Algorithms can generate other algorithms; not only precise, identical, multiple copies of themselves but also structures text (i.e. code) that when executed will behave as an algorithm. In fact the process of composing an algorithm is also an algorithm in itself, that is, the algorithm that created the algorithm. This self-referential property (which can be referred to here as meta-algorithm, i.e. the algorithm of an algorithm)

Algorithms in Design- What will an algorithm do to any design?

Defining **Algorithm**

An algorithm is a computational procedure for addressing a problem in a finite number of steps. It involves deduction, induction, abstraction, generalization, and structured logic. It is the systematic extraction of logical principles and the development of a generic solution plan. Algorithmic strategies utilize the search for repetitive patterns, universal principles, interchangeable modules, and inductive links. The intellectual power of an algorithm lies in its ability to infer new knowledge and to extend certain limits of the human intellect. An algorithm may be compared to the steps in a recipe; the steps of gathering the ingredients, preparing them, combining them, cooking, and serving are algorithmic steps in the preparation of food. Obviously, the number, size, and quality of ingredients, the sequence and timing of events, as well as the serving and presentation of the final product are key factors to a recipe. Theoretically, an algorithm is the abstraction of a process and serves as a sequential pattern that leads towards the accomplishment of a desired task.²⁹

An algorithm is not only a computer implementation, a series of lines of code in a program, or a language, it is also a theoretical construct with deep philosophical, social, design and artistic repercussions.

Imagine a few drops of water about to freeze. The endless variety of crystal shapes that emerge in that moment of crystallization became an obsession for one self-educated farmer from Vermont, Wilson Bentley, who spent a lifetime photographing snowflakes. For the forty-five years leading up to his death in 1931, through a painstaking process that involved brutal weather and fussy equipment, Bentley proved that no two snowflakes are alike by documenting 5,381 individual crystals falling behind his farmhouse. For him, the beautiful six-sided symmetry of every crystal was evidence of both the character of the cloud it came from, its altitude, electromagnetism, and temperature, as well as the rules inherent to the water molecule. Since science had not unravelled a working model of the atom yet, it was this last detail-the water molecule itself with its attractions and repulsions- that led Bentley to his exasperation in an article for Technical World in 1910: "What magic is there in the rule of six that compels the snowflake to conform so rigidity to its laws?" Beneath Bentley's exasperation is a yearning for the algorithmic; the rule of six is evidenced not only in the fact that all snowflakes are six-sided but right down to the molecules and the way they bond with each other, ultimately describing a molecular relationship between two hydrogen atoms and one oxygen atom. The rule of six unlocks his collection. It is a binding rule of transformation, an algorithm that connects the movement from "six" to "no two are alike".³⁰

²⁹ Algorithmic Architecture – Kostas Terzidis.

³⁰ Tooling – Aranda/Lasch

Algorithm and Meta algorithms are important in design for at least two reasons.

First, like algorithms, design can be seen as a set of procedures that lead stochastically towards the accomplishment of a goal. In studying the articulation of algorithms one may be able to discern the similarities with design. While such a study may lead to the development of procedures that may be useful in design, more importantly, it may reveal certain clues about design as a mental process. This possibility opens up a more intricate relationship between design and algorithm than has been previously possible. Rather than using algorithms to copy, simulate, or replace manual methods of design (while perhaps desirable), instead they can be studied as methodologies that operate in ways similar, parallel, or complimentary to that of the human mind.

Second, along the lines of *homo faber homo fabricatus* (i.e. we make a tool and the tool makes us), algorithms can be seen as design tools that lead towards the production of novel concepts, ideas, or forms, which in turn, have an effect in the way designers think thereafter. That way of thinking is incorporated in the next generation of tools that will, in turn, affect the next generation of designers, and so on.

It is a common belief among architects and designers that the mental process of design is conceived, envisioned, and processed entirely in the human mind and that the computer is merely a tool for organisation, productivity, or presentation. Whatever capabilities a computer may have it lacks any level of criticality and its visual effects are nothing but mindless connections to be interpreted by human designer. It is a common belief that, at best, the computer can serve merely as a processor of information provided as data by the designer and as code by the programmer outputting simply the results of data processed by algorithms. What makes this process problematic is the fact that contrary to common belief algorithms can produce results for which there is no intention or prediction thereof of their behaviour. Further, algorithms can also produce algorithms that also are not connected to the intentions or predictions of the original code. This structural behaviour resembles in many ways Dadaist poetry, or Markov processes. In those cases, an algorithm functions as a string rewriting system that uses grammar-like rules to operate on strings of symbols in order to generate new strings of text. While the syntax of the resulting text may be consistent with the grammatical rules, the meaning of the resulting text is not necessarily associated semantically with the intentions of the original code. For instance, the introduction of randomness in the arrangement of text can produce results that are unpredictable, but also accidentally meaningful. Unpredictability is, by definition, a disassociation of intention. But unlike chaos, a random rearrangement of elements within a rule-based system produces effects that, although unpredictable, are intrinsically connected through the rules that govern that system. In the field of design, similarities may exist on formal, visual, or structural levels. Computational rearrangement of formal rules that describe, define, and formulate a certain style can produce a permutation of possible formal expressions for that style.³¹

Vier Maurer saßen einst auf einem Dach.
Da sprach der erste: "Ach!"
Der zweite: "Wie ists möglich dann?"
Der dritte: "Daß das Dach halten kann!!!"
Der vierte: "Ist doch kein Träger dran!!!!!"
Und mit einem Krach
Brach das Dach.

‘So, so!’ – Kurt Schwitters

English translation:

Four masons once sat on a roof
Then the first: "Ah!"
The second: "How is it possible then?"
The third: "The fact that the roof can hold!"
The fourth: "Is not a carrier to it !!!!!"
And with a crash.
Broke the roof.

An example of Dadaist poetry³²

³¹ Algorithmic architecture

³² <http://members.peak.org/~dadaist/English/Graphics/soso.html>

Algorithmic techniques

Understanding algorithms in design through some techniques such as

- Spiralling
- Packing
- Weaving
- Blending
- Cracking
- Flocking
- Tessellations/Tiling

Tessellations/Tiling is taken in depth for understanding as Voronoi algorithm basically deals with tessellations as an output

A brief description of algorithms with reference to all other techniques:

Spiralling

An algorithm which defines a start point and growth or emergence from that point if controlled by an order or logic will result into a basic effect of spiralling. Patterns will be drawn in a loop but always with a higher order added.

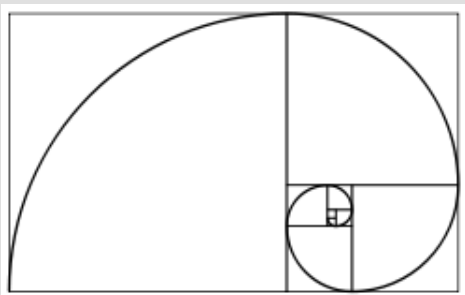


Figure 15 – The golden mean

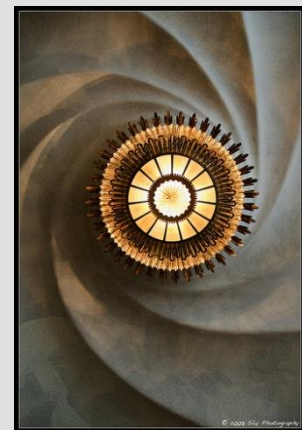


Figure 16 – Casa Batllo ceiling

Packing

Stability through adjacency

Packing is a powerful organisational method in which an element's position in regard to its neighbours is determined by certain rules – not too close, no overlapping etc.

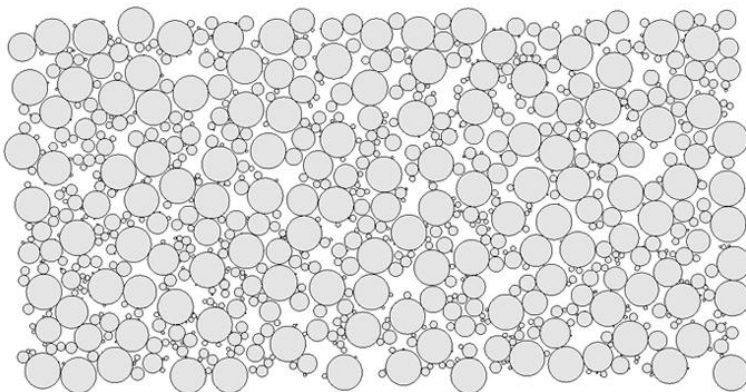


Figure 17 – computer generated image of circles in close packing.

Weaving

Production of strength by combining two weak systems in a reciprocal pattern.



Figure 18 – A basket weave technique.

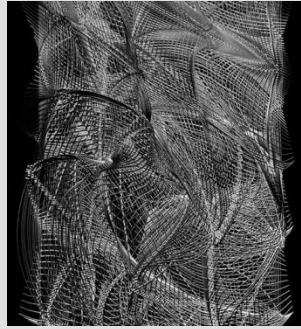


Figure 19 – Variable frequency weaving.

Blending

Fundamental technique in the act of negotiation.

Cracking

Following the rule of self-similarity, cracking gives the sense of larger whole.

Flocking

Flocking finds order through Entropy

Chapter One.
Introduction-Voronoi Geometry

A Brief History
2D Voronoi
Centroidal Voronoi Tessellation
3d Voronoi
Other types of Voronoi

Chapter two.
Voronoi Geometry in nature

Dragonfly wing
Soap Bubbles

Chapter three
Understanding Algorithms as a tool in Design

Algorithmic Techniques

Spiralling
Packing
Weaving
Blending

Chapter four
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Cracking
Flocking

Understanding Voronoi algorithm as a tool for tessellation

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Chapter seven
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C_Wall
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"Patterns are magical marks, open pictures for the mind to travel thought."

Akin to the visible spectrum in electromagnetism that colours our imagination, patterns offer structure, to the realm of ideas. In this region metaphor is at one end, concrete realisation at the other. A contemplation of one leads to the other.



To give meaning to pattern we impose scale, a molecule-map or a road-map – but pattern is invariant; a fix at one level raising a speculation at another. Mediation on pattern suggests connectivity – and in here are the archetypes of structures, the networks of configuration

The diagram acts as a catalyst, and continues its private haunting, attacking certainties. Whatever is seen in a reduced reality seems to have ghost in another dimension – the mystery denies dogma as a natural state. Mind seems to have its own subversive structure; to find out I run the templates³³ – CECIL BALMOND

Tessellation/ Tiling

Tessellations or Tiling assembles a patterned tectonic

Tessellation or tiling along with repetition and modularity provides the issue of adjacency.

Voronoi acts as a recipe for tiling/tessellations.

There are Voronoi algorithms which are not mentioned till this point in this dissertation as the concern here is to understand Voronoi diagrams diagrammatically. But to have an evident description of Voronoi diagram as algorithms some of the algorithms are shown

$$Vor(p, S) = \{x \in \mathbb{R}^2 \mid \|xp\| \leq \|xp'\| \forall p' \in S\}$$

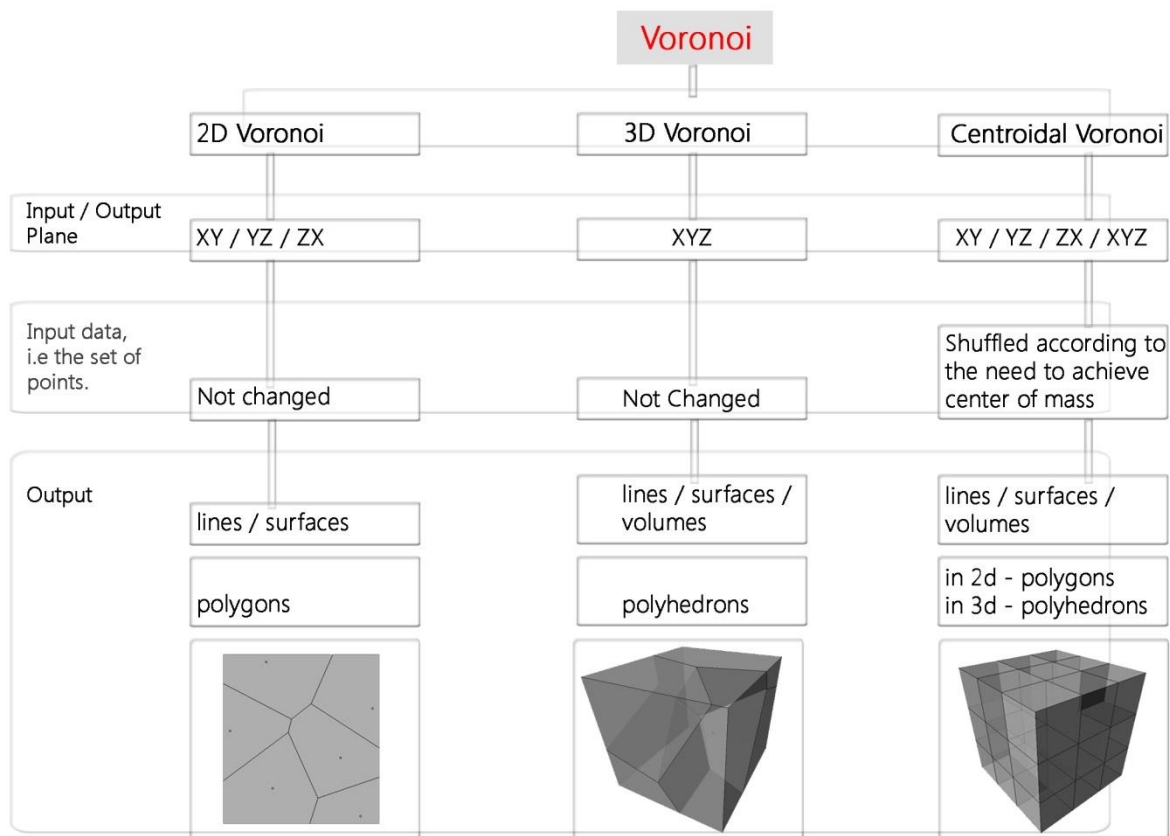
A More complex one is

$$h(q) = \sum_{p \in S} \frac{\text{Area}(Vor(q, S \cup \{q\}) \cap Vor(p, S))}{\text{Area}(Vor(q, S \cup \{q\}))} \cdot h(p)$$

Tessellation/Tiling in Design

- Repetition
- Modularity
- Regularity
- Adjacency
- Transformation

³³ Informal – Cecil Balmond.



“One of the emerging spatial paradigms is that of the network as a system of interrelations between dissipative processes and aggregative structures that shape new spatial patterns and protocols”
Kolatan/Macdonald

Network - definitions

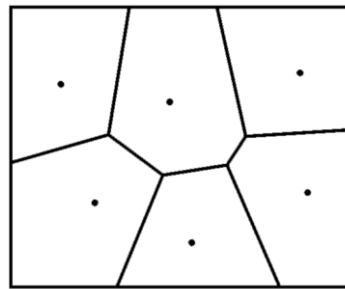
- (a) A fabric or structure of cords or wires at regular intervals and are knotted or secured at the crossings.
- (b) 1. an interconnected or interrelated chain, group, or system;
2. a system of computers, terminals and databases connected by communications lines³⁴
- (c) 1. The joining up of pattern, with function – a star map – a road map – a molecule³⁵

³⁴ Merriam Webster’s Dictionary – Index Architecture . Bernard Tschumi and Matthew Berman.

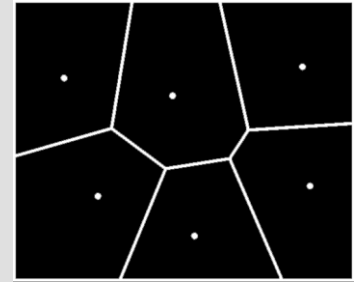
³⁵ Informal - Cecil Balmond with Jannuzzi Smith

The Voronoi Diagrams as networks

Given a set of random variables or parameters or conditions in design, here through Voronoi diagrams, output is measured with respect to boundary and domain.



BOUNDARY



DOMAIN

Framework for analysis.

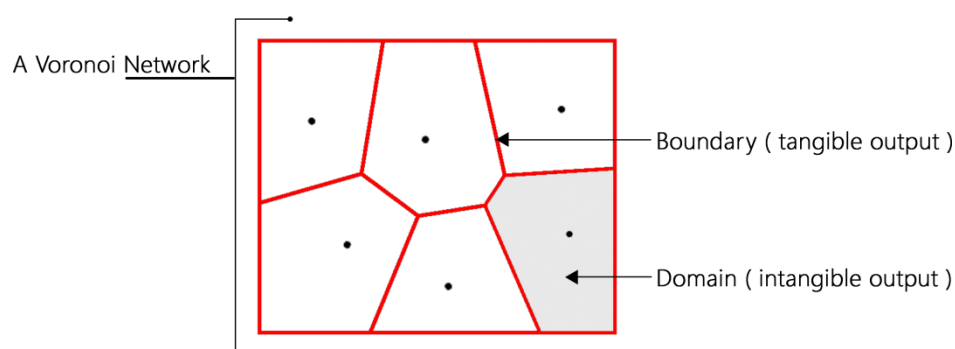
In order to understand the output of Voronoi tessellations in design, these are some defined design variables that are taken into consideration.

- Materialising information.
- Material Optimization.
- Structural Performance.
- Variety Production.

These variables are interrelated but if individually taken as an initial operator in a design process, application of Voronoi algorithm can be a solution.

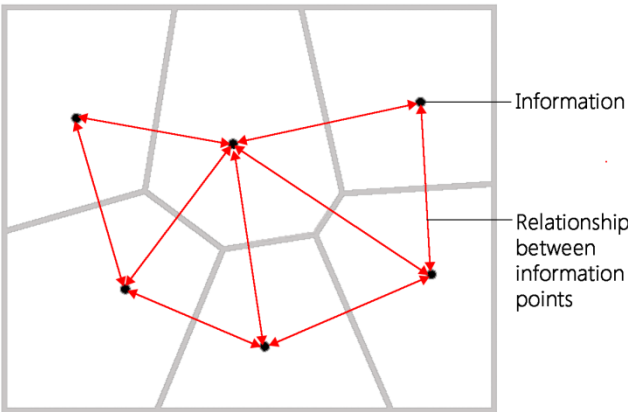
Developing an understanding between Voronoi algorithm (diagrammatic representation) and the defined variables.

Based on the output of Voronoi algorithm as networks it can be categorized into boundary and domain. If we analyse it further on the basis of its output, it can also be defined as a tangible output and an intangible output. Both holds an equal value in terms of output and can help in understanding the relation with design variables and Voronoi algorithm.



Understanding
First defined variable, which is

Materialising Information



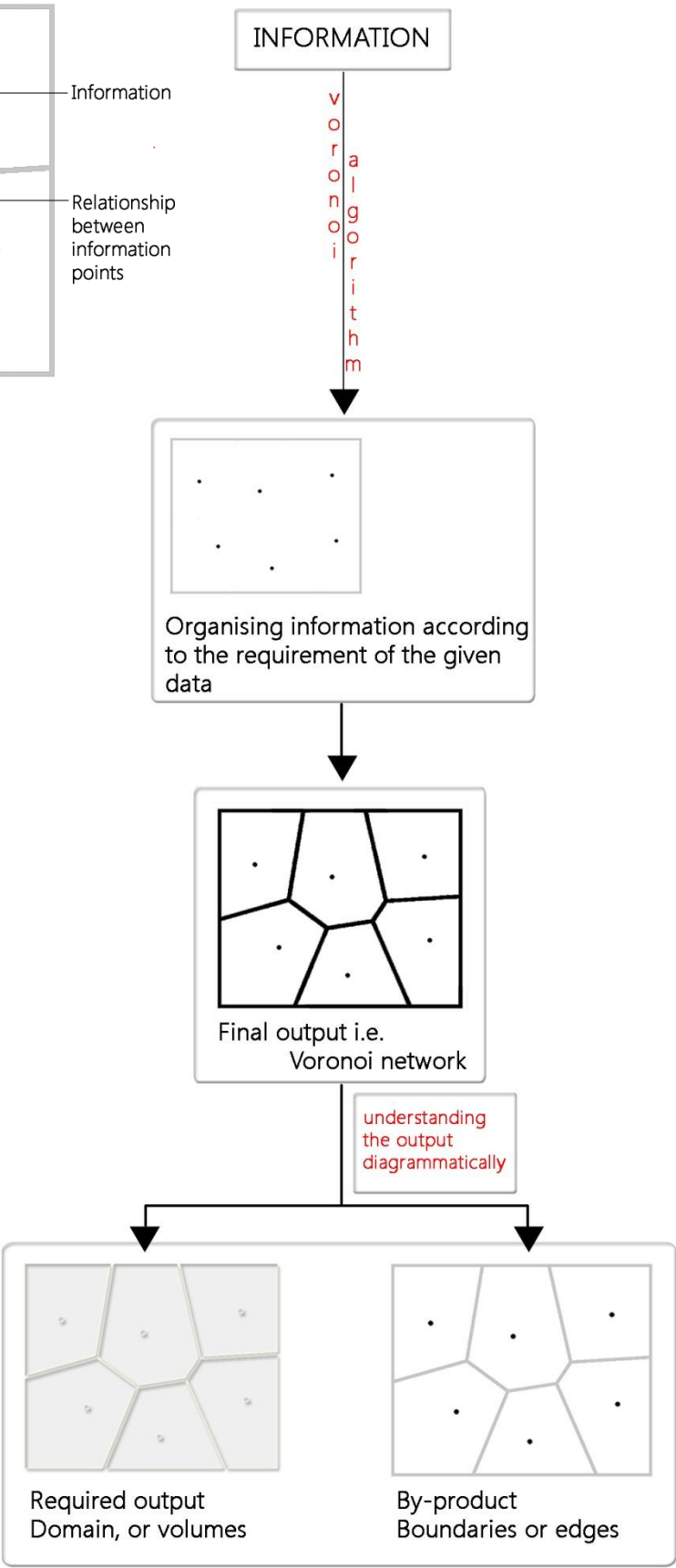
Design Variable of
materialising information has
been categorized under as

A domain based approach.

The given set of random
points is the information.

Its relationship to each
other (i.e. the red lines in the
diagram) cannot be altered or
repositioned. Thus the use of
Centroidal Voronoi tessellation
might not appear to be a useful
tool, as it tends to readjust the
given data points to make it the
centre of mass of their
respective domains.

In dealing with this
particular design variable, the
output matters in terms of the
volumes achieved. So the
boundaries, which can be the
tangible output behaves as a by-
product. Boundaries are
considered as a by-product
because if this step in a design
process is not the final step
which will not lead to any
tangible output than for such
consideration (design variable)
the volumes or domains are
important in terms of an output.



Design variable of **material optimization** has been categorised as

A boundary based approach.

From the study of dragonfly wing and soap bubbles, in which both deals with material optimization and structural performance it appears quite difficult to categorize material optimization as a variable, resultant of a boundary based approach.

But for a diagrammatic understanding of Voronoi networks and the sequential or step wise approach of algorithms, material optimization can be categorised as a boundary based approach.

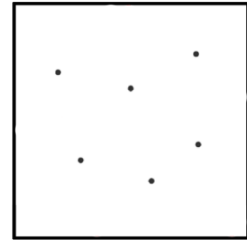
The output of such a variable is always measured with respect to material, thus it has to be a tangible output which directly puts it into a category of boundary in such analysis.

While considering material optimisation as a design variable, understanding from the study of dragonfly wing and soap bubbles clears that in such condition Centroidal Voronoi tessellation appears to be more functional than Voronoi of random points. The data points also have a flexibility of shuffling themselves to provide minimum length boundaries (for example chart ())

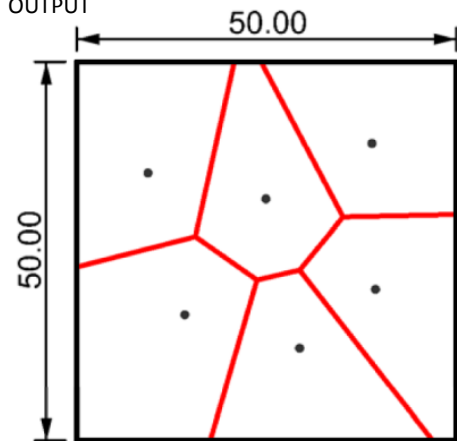
From the given chart it clarifies that this parameter of design variable clearly deals with boundary as an output. The given set of points and the domains are flexible in nature and in a way help the boundaries to achieve optimization in a particular state.

To partition six random points inside a square of 50 x length of lines

Given set of random points

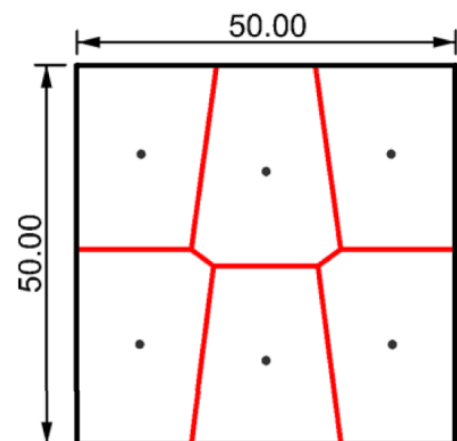


OUTPUT



For this output Voronoi algorithm for random points is applied, the original point set is not changed.

The total length of all partition lines (i.e., the red lines) is **153.1721 mm.**



For this output Centroidal Voronoi algorithm for random points is applied, the original point set is changed for all points to become the centre of mass of their respective cells.

The total length of all partition lines (i.e., the red lines) is **147.9051 mm.**

“Science works always to achieve general theories that unify knowledge. Every specific natural event, to be scientifically satisfying, must ultimately be related to a general formulation. Engineering, in contrast, works always to create specific objects within a category of type. For Each design, to be technologically satisfying, must be unique and relate only to the special theory appropriate to its category.”³⁶

David. P. Billington. The Tower and the Bridge. The New art of Structural Engineering.

Considering the design variable of structural performance, it cannot be analysed without the context or related parameters.

Under which condition should we analyse the structure’s performance?

For example,

- To create the tallest tower, for such structure the related parameter is of height.
- To achieve maximum cantilever, for such condition the related parameter is of the length which is cantilevered.
- To create an earthquake proof building, for such the aspect of structure holding itself under the amount of impact is the related parameter.

Now for our analysis, categorisation of Voronoi based structures under domain based approach or boundary based approach is quite not logical. It is not logical because if such process analysed in terms of tangible and intangible outputs, the boundary and domain holds equal importance even in terms of processing in real as well as virtual medium.

In fact this can be considered as

A network based approach, instead of categorizing them furthermore.

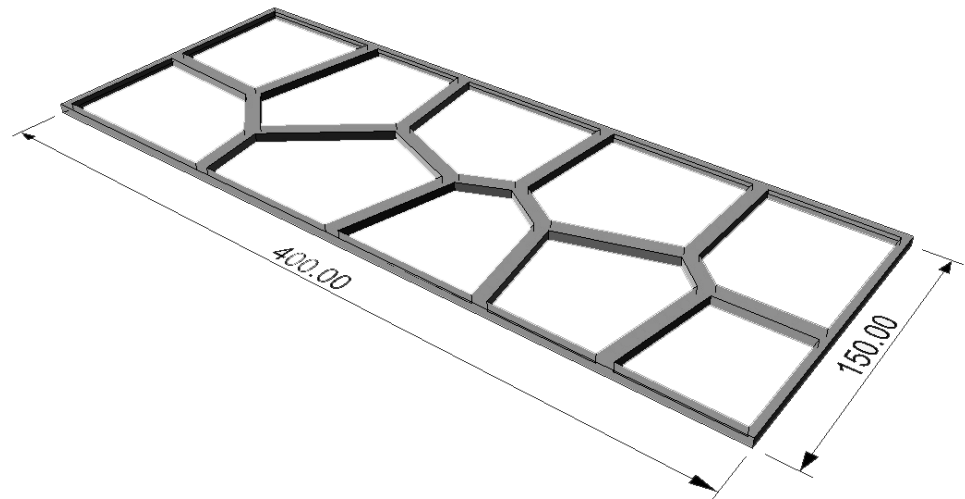
Networks when measured with respect to structural performance, as said earlier need a variable through which it can be measured. For doing so, a small exercise is done on two Voronoi networks with weight as a variable to measure their structural performance.

While doing a structural simulation in a plugin called SCAN and SOLVE in rhinoceros software, the fixed parameters for both the frames were:

- Width – 150 mm.
- Length – 400 mm.
- Thickness – 5 mm.
- Volume- 58246.9759 (+/- 1e-05) cubic millimetres
- Number of data points to generate Voronoi network – 10.

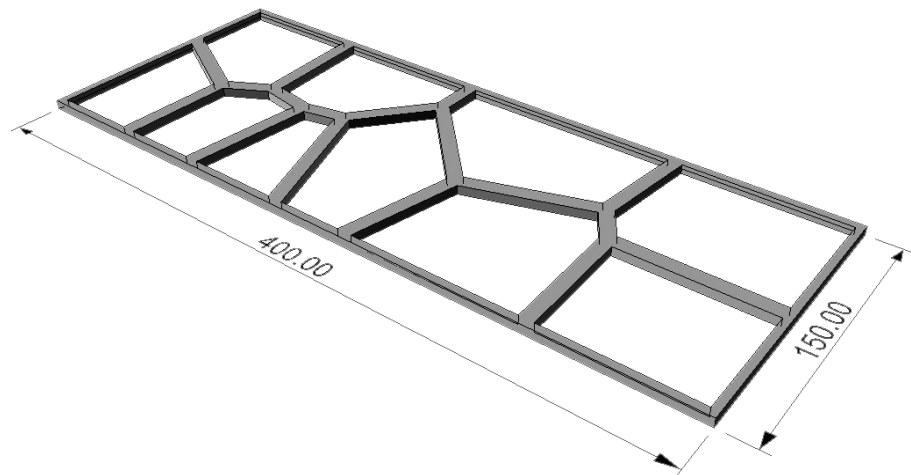
³⁶ AD – Versatility and Vicissitude.

Frame A:



Dimensions in mm.

Frame B:



Dimensions in mm.

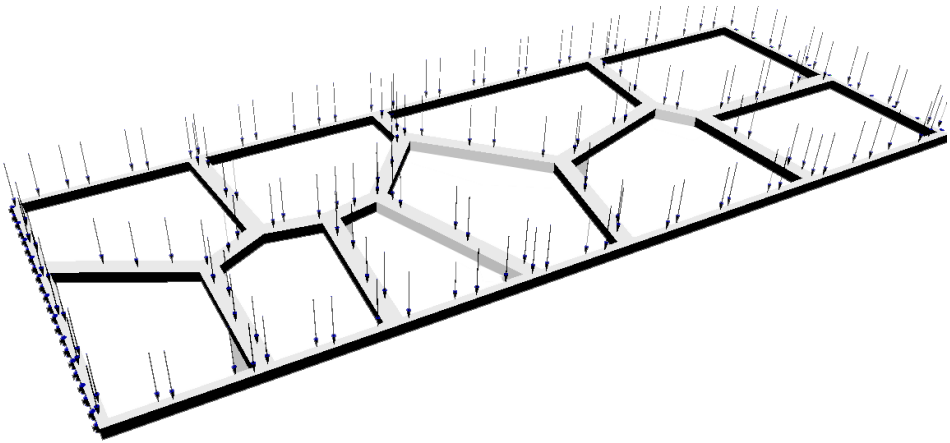
Now as the volume of the frames is same, when we convert them into any material their respective weights will be same.

For this exercise, aluminium 1050-O is selected as a material. Thus the weight of both the frames will be same.

Material Properties

Property	Value
Description	Aluminium 1050-O
Density	2.705e-09 Mg/mm ³
Elastic Modulus	69000 MPa
Poisson Ratio	0.33
Default Failure Criterion	von Mises
Tensile Yield Strength	28 MPa

Now in order to understand structural performance with respect to network based approach, pressure is applied as a variable on both the frames in this manner (figure()).



The shorter faces are held as restrain faces and the pressure is applied on the top of both the frames.

Load Summary

Description	Type	Definition
Load	Pressure	0.004 MPa
Body Load	Gravity	{0,0,-9806.65} mm/s ²

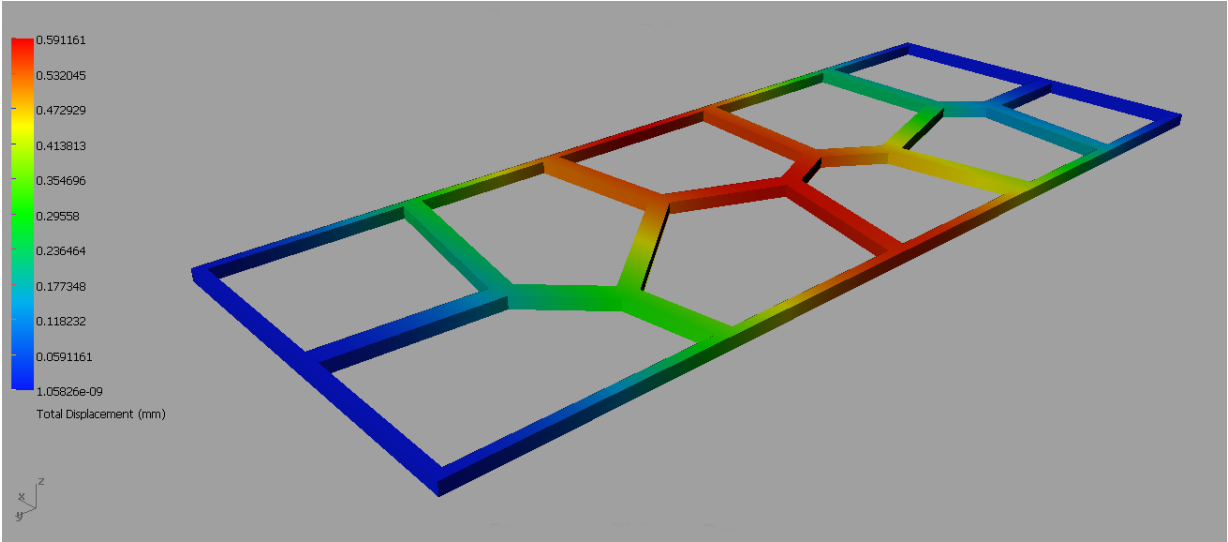
Restraint Summary

Description	Definition
Restraint	X-Fixed-Fixed-Fixed

Units

Quantity	Unit
Length	mm
Mass	Mg
Force	N
Time	s

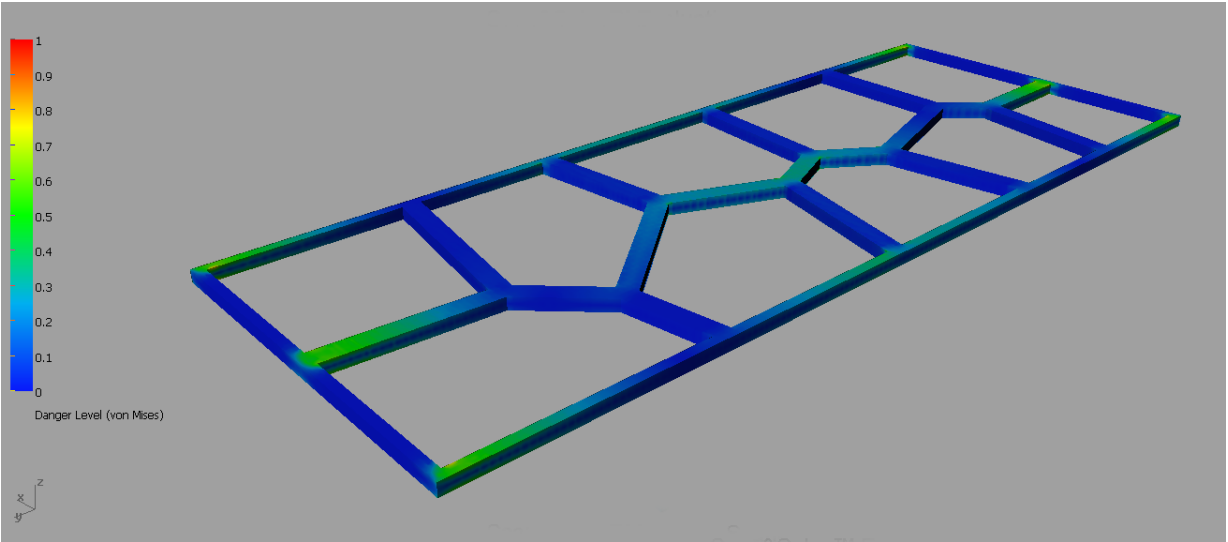
Simulation Results for frame A



Displacement Summary

	Amount	Location
Minimum	1.05826e-09 mm	{501.932,-3443.52,2.5}
Maximum	0.591161 mm	{565,-3244.45,2.5}

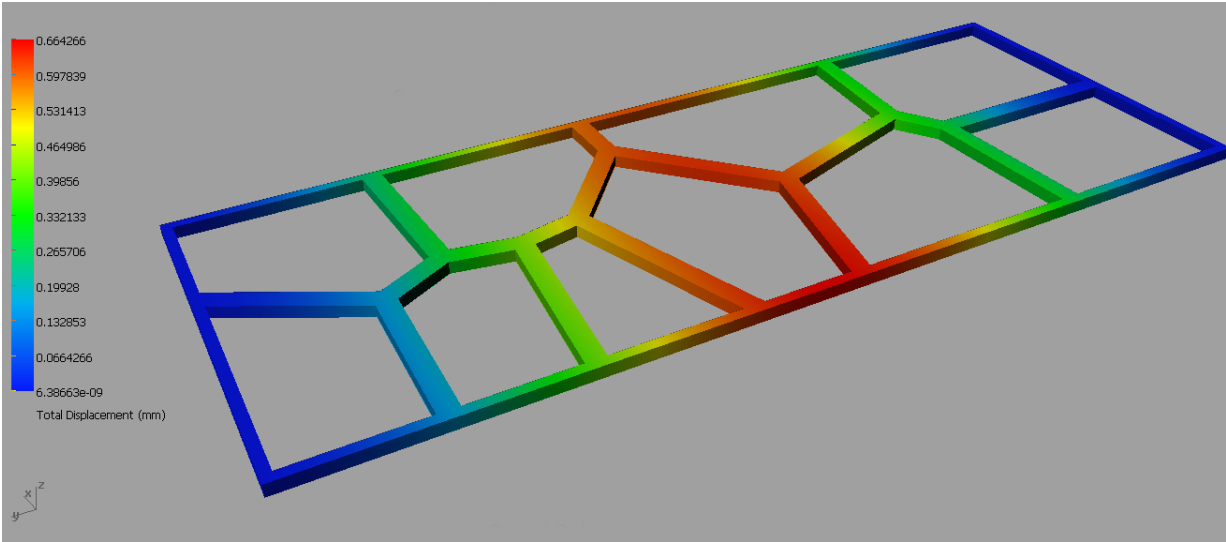
Danger Level



Danger Level Summary

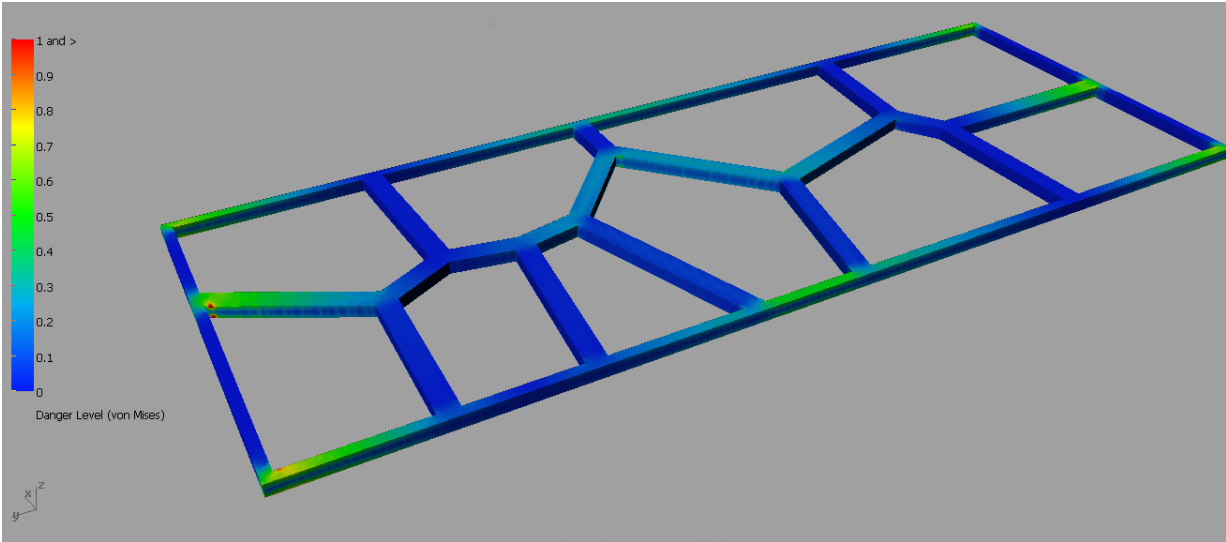
	Amount	Location
Minimum	2.08526e-05	{437.159,-3443.52,2.5}
Maximum	0.828498	{561,-3436.87,5}

Simulation Results for frame B



Displacement Summary

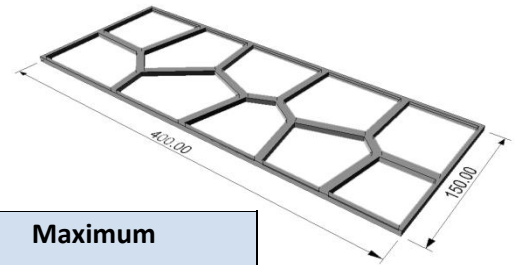
	Amount	Location
Minimum	6.38663e-09 mm	{734.432,-3429.52,2.5}
Maximum	0.664266 mm	{685,-3224.89,2.5}



Danger Level Summary

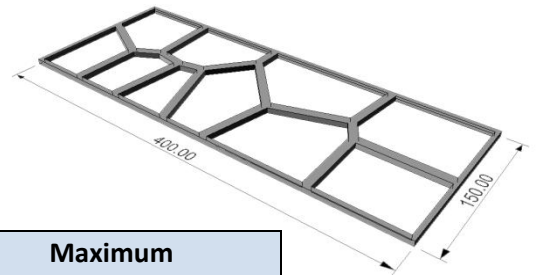
	Amount	Location
Minimum	3.98375e-05	{787.928,-3426.02,0}
Maximum	Criterion Limit Exceeded	{774.613,-3035.6,0}

Results for Frame A:



	Minimum	Maximum
X-Displacement	-0.000835654 mm	0.00083696 mm
Y-Displacement	-0.010847 mm	0.0108412 mm
Z-Displacement	-0.591161 mm	6.06889e-05 mm
Total Displacement	1.05826e-09 mm	0.591161 mm
Von Mises Stress	0.000583873 MPa	23.198 MPa
Max. Principal Stress	-6.67715 MPa	29.6461 MPa
Mid. Principal Stress	-9.57162 MPa	9.55139 MPa
Min. Principal Stress	-29.6171 MPa	6.55532 MPa
Danger Level (von Mises)	2.08526e-05	0.828498

Results for Frame B:



	Minimum	Maximum
X-Displacement	-0.00173549 mm	0.000998884 mm
Y-Displacement	-0.0117144 mm	0.01192 mm
Z-Displacement	-0.664266 mm	8.79628e-05 mm
Total Displacement	6.38663e-09 mm	0.664266 mm
Von Mises Stress	0.00111545 MPa	50.9298 MPa
Max. Principal Stress	-36.382 MPa	61.7758 MPa
Mid. Principal Stress	-40.0336 MPa	23.9957 MPa
Min. Principal Stress	-89.0394 MPa	23.0318 MPa
Danger Level (von Mises)	3.98375e-05	Criterion Limit Exceeded

Results prove that under such condition frame A will exhibit better performance when compared to frame B.

While understanding and performing this exercise, the approach was compulsorily a network based approach. It is not logical to proceed when any of the one component i.e. either boundaries or domains are considered an individual entity.

Thus, while understanding structural performance as a design variable the structure has to be realised as a network, in which the boundary and the domain both are equally respondent. None of them can be taken into consideration separately because of its responsive character.

Understanding

related defined variable, which is **Volume/Weight Ratio.**

This variable can be considered as a subset of structural performance. For example, Polyhedrons when analysed with the criterion of volume/Surface area ratio, surface area directly outputs the parameter of weight.

In the later chapter this is discussed in detail with the reference of tetrakaidecahedron and Weaire Phelan structure.

What is the most efficient way to divide space into cells of equal size with the least surface area between them?

- Lord Kelvin

For this analysis when such variable is taken as a primary concern for a design process, it is much similar with the analysis of structural performance, i.e. it will not be logical to categorise it into a domain based or a boundary based approach.

Volume is a measurable factor of a domain and weight is of the boundaries. So while taking Volume/Weight ratio it is not possible to take just one of them into consideration. Thus it has to be a network-based approach.

Definition – mass production.

Definition – Mass customization.

Mass customization aims at producing goods and services catering to individual customer's needs with near mass production efficiency (Tseng and Jiao 2001).³⁷

Defining Variety production –

Variety production as interpreted here, lies somewhere in between mass production and mass customization, It means that some product or design belong to a particular family but it is still different for every individual. It deals with customizing the design according to need or information provided by individual client.

Algorithms are a means to translate such requirements into a design process and still can allow it follow a general language.

³⁷ <http://www.mass-customization.de/download/mchandbook2010.pdf>

Understanding Tessellations

A Brief History

Tessellation can be seen in the world over from mosaics in ancient Rome and those of the Byzantine Empire to the screen walls in Islamic architecture or the stained-glass windows in Gothic cathedrals.

These decorative surfaces were used to filter light or view, define space, or convey symbolic meaning through an abstracted notational language, much the way tessellated surfaces are used in design today. Because these early examples were handcrafted, overall patterns were typically achieved by laboriously assembling many small pieces into a coherent design or image. It was a time-intensive enterprise, but this aggregative technique fostered vast figural, imagistic, tonal, and geometrical variation.

Spatial Patterns of the Present

Today's spatial design pattern morphologies are mainly digital/parametric or Postmodern reworking of ancient patterns (like waves) or new ones (like DNA) found or simulated with new and emerging visualisation and design technologies. Among these we find patterns of soap bubbles, Fibonacci series, hydrological and vascular systems, protein folds, cellular automata, attractors, force fields, Sierpinski cubes, skins, moirés, knots, messes, fractals, networks, swarms/flocks, atoms and molecular structures (including crystals and quasi-crystals), fluid and gas/smoke/meteorological forms and dynamics, architextiles,²⁴ viruses and micro-organisms, blobs, Voronoi cells, Linden Meyer systems, light, fire, landscapes/geology/geography, rhizomes and various hybrids and permutations of these.

In many of these designs, the crucial innovation is either technologically enabled patterns and/or patterns as fields, membranes, complex surfaces, deep structures or formless ambient environments and affective atmospheres.²⁵ The most technically sophisticated are designed using genetic algorithms, and parametrically with software programs such as Grasshopper, Generative Components, Processing and L-Systems.³⁸



Figure 20-Mosaics in Ancient Rome



Figure 21-
Mosaics in
Byzantine
Empire



Figure 22- Screen Walls in Islamic
Architecture



Figure 23 – Stained Glass Window in Gothic
cathedral.

³⁸ Patterns of Architecture – Architectural Design

Tessellations and digital technologies

Digital technologies have revitalized the design world's interest in patterning and tessellation because they afford greater variation and modulation through nonstandard manufacturing, even as they provide an inherent economy of means. Working digitally enables movement from one representational format to another—for example, from digital model to vector-line file to manufacturing method. This series of translations allows for a more fluid fabrication process while significantly reducing the labour associated with taking one type of design medium and turning it to another.

While mosaics, brick walls, stained-glass windows, and panelised facades can all be considered tessellated, the term can also refer, in digital design, to approximating surfaces, often singly or doubly curved, with polygonal meshes. Curved surfaces are typically far more complex and expensive to construct than flat ones, and tessellation offers a way to build smooth form using sheet material.

Tessellation or tiling is becoming increasingly relevant to designs as designers strive to make large, often complex forms and surfaces with standard-size sheet materials. Whereas in modern architecture, tessellation has been the result of using industrialized products such as ceramic tiles, siding, and bricks, it can now be created from nonstandard units.

There seems also to be a tendency towards polygonal tessellations in contemporary envelopes – including PTW's Beijing Water Cube (2007), Future Systems' Selfridges department store and FOA's Ravensbourne College of Design and Communication in Greenwich, London (due for completion in 2010) – that oppose the Cartesian grid division of the late Modern screens. This tendency is first made possible by the release of the envelope from structural and environmental control functions.

Polygonal geometries have additional performances: for example, a hexagonal tiling has less joint length than a rectangular tile of the same area. If the contemporary envelope has more stringent requirements in terms of insulation and security performance, a polygonal tessellation will provide a smaller joint length per surface unit than rectangular grids, so this tendency may even be driven by a contemporary desire for sealed, immunising atmospheres. But it is certainly enhanced by a faciality that is no longer structured in planar, vertical and discrete faces, as some of these envelopes explore differential geometries of the surface: the construction of bubble envelopes is not possible using a Cartesian tessellation.³⁹



Figure 24 -The Geodesic Dome (1967) in Montreal by Buckminster Fuller comprises three quarters of a geodesic sphere. The outer Hull is made out of triangles and is linked to the inner hull consisting of hexagons.

³⁹ AD – The Patterns of Architecture – Mark Garcia (Garcia)

Designers have certainly made intricate patterns from conventional materials such as brick and masonry, but tiling has found new potency in the arena of digital manufacture, which is unique to modulate. Design, and build custom panels. Rather than rely on what is commercially available designers can, using digital manufacturing techniques, cut pieces from larger stock in multiple differentiated sizes and shapes.

There are two primary ways to model three-dimensional forms digitally:

- NURBS
- Meshes

A single project will often be defined in both formats at different stages of the design process.

NURBS

Non Uniform Rational Basis-Spline.

NURBS modellers build smooth curves and surfaces.

Meshes

Mesh modellers use polygons and subdivisions to approximate smooth surfaces. Polygonal meshes, usually made up of triangles and quadrilaterals, are the most widely used; subdivision surfaces use a secondary, more complex algorithm to approximate curvature. Both create additional vertices, edges, and faces that break the surface into tiles. Hence a polygon or subdivided mesh is a tessellated surface.

Depending on the resolution of tessellation, approximated surfaces can be smooth and precise, or faceted and crude. Although it may seem desirable to be highly accurate all the time, this is not necessarily the case. It is often unnecessary to over tessellate a form; it results in a cumbersome and heavy computer model and often in unbuildable form.

When evaluating tessellation strategies, if the aim is to calibrate the initial form with a constructional system, one may better determine the size and resolution of the tiles relative to overall geometry and design intention, and with regard to final building materials and fabrication processes.

Buckminster Fuller's geodesic domes are early examples of such approximation. Fuller's pursuit of lightness and engineered efficiency is epitomized in the domes he designed for mass production and ubiquitous use. Though they did not catch on as he intended, many of his domes were built in a variety of tessellated patterns. In every case, the spherical shape is redefined as a pattern of triangles or hexagons that provides structural stability and resists shape deformation. The elegance of the structure is dependent on the uniform curvature of the dome.

In the geodesics, every strut, opening, and joint detail is identical. While this uniformity contributes to ready constructability and overall material efficiency, it is unrelenting in terms of form.

The past decade and half has seen the rapid acceleration of discretization – the digital definition of surface as a coordinate set of discrete parts – as a digital and material practice. Not only is it a logical way to describe and build non-orthogonal forms, it is a method that enables designers to modulate and gradate surface and skin.

Peter Macapia of labDORA has suggested that one fascination with such surface systems may be *“in part of consequence of the changing nature of how we see architecture no longer as a point or an object in space, but rather as a function, a function of grids, of networks, of gradients.”*⁴⁰

⁴⁰ Digital Fabrications – Lisa Iwamoto

Close Packing in 2D.

Tessellation

Tessellation of a plane.

A Plane tessellation is an infinite set of polygons fitting together to cover the whole plane just once, so that every side of each polygon belongs also to one other polygon.⁴¹

Regular Tessellations.

A regular tessellation is a pattern of congruent regular polygons filling the whole plane, on which all vertices of the tessellation are surrounded alike (uniform).

This can be simply explained by pointing out that in order to subdivide the plane with polygons, the angles around each vertex must sum to 360° . Any polygon with face angles greater than 120° will not be capable of forming a tessellation.

There are only three possible regular tessellations.

- Tessellations of triangles.
- Tessellations of squares.
- Tessellations of hexagons.

Semi regular Tessellations.

This is a second class of planar partitions. This class requires that all polygons be regular, and that all vertices be congruent, but permits more than one kind of polygon. There are only eight possible cases of semi-regular plane tessellations.

Tessellations with regular polygons.

If the plane is to be filled exclusively with regular polygons, but do not require that all vertices be surrounded by equal angles, a new class of tessellations can be constructed in which an infinite number of patterns are possible.

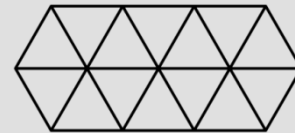


Figure 29.a –
Triangular
tessellations.

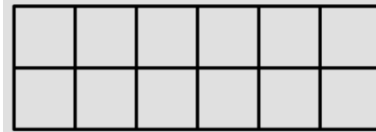


Figure 29.b –
Square
tessellations.

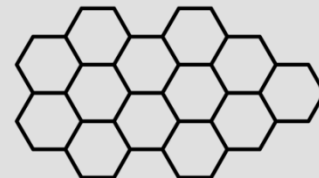


Figure 29.c –
Hexagonal
Tessellations.

Diagram 29 – Regular
Tessellations

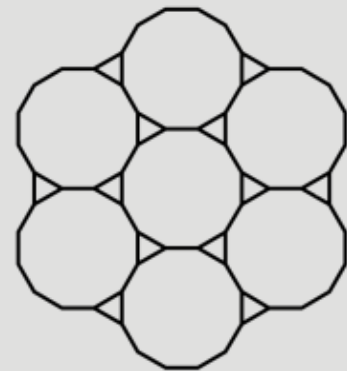


Diagram 30 –Semi - Regular Tessellations

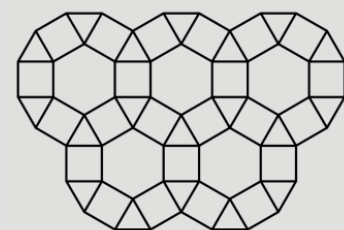


Diagram 31 –Tessellations with regular polygons

⁴¹ Structure in nature is a strategy for design.

Dual Tessellations.

A dual network is formed by joining the centres of each polygon to all neighbouring polygons through the shared edges. Only one of the regular and semi-regular plane tessellations is dual to itself – namely, the square grid.

The dual network always forms polygons which are the domains of the vertices, i.e. polygonal domains will have the same number of edges as there are edges meeting at the vertex it encloses. Any network of a plane tessellation with congruent vertices will have a unique plane filling domain formed by its dual network.

Consider the partitioning formed by closest packed circles; although each circle by itself is very economical (i.e., it encloses maximum area for its given perimeter) small concave triangles are formed between circles. Now, concave triangles match the least area with the given circumference. Consequently we can say that, considering the entire plane, circle packing is not the most economical system. However let us allow the circles to change their shapes such that fill up the concave triangles, forming hexagons. This becomes the most economical method of partitioning a surface into equal units of area.

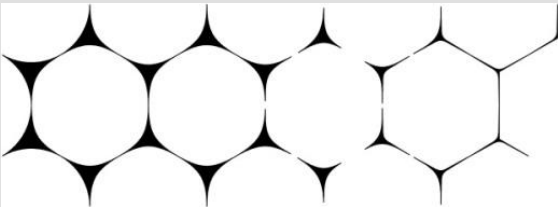
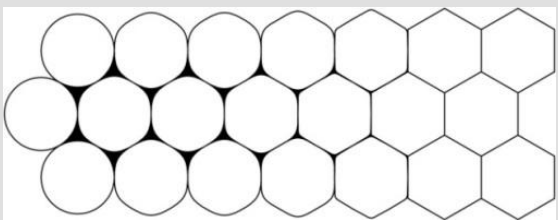


Diagram 32 –Tessellations with regular polygons

Close Packing in 3d.

“Space filling”

Space filling means the combining of like or complimentary bodies in a three-dimensional packing continuously repeated, in such a way that there is no unoccupied space.

Close packing of Solids.

Closest Packing is a structural arrangement of inherent geometric stability that finds expression in the three-dimensional arrangement of polyhedral cells in biological systems as well as in the dense arrangement spherical atoms in the structure of certain metals.

If the centres of closest packed equal spheres are joined, a three dimensional arrangement of equilateral triangles is formed. If the centres of closest packed polyhedral cells in a biological structure are joined through shared faces, a triangulated configuration will also result.

It can be readily seen that the principle of closest packing is equivalent to that of triangulation, and it is well known that triangulated frameworks exhibit inherent geometric stability. Such properties enable framework structures to be built without moment joints, insuring axially loaded members; and this in turn results in high strength-per-weight minimum-energy structures. Planar and domical structures have taken advantage of triangulation for a number of years; the first truly three-dimensional triangulated structures were probably Alexander Graham Bell's tetrahedral kites and space frames. The principle of closest packing/triangulation is one of remarkable universality. It operates independently of scale or materials, with the same energetically conservative effect.

Whether at the molecular level, the cellular level, or at the man-made structural level, its inherent stability always establishes a condition of minimum potential energy.⁴²

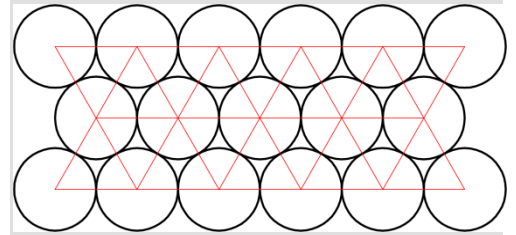


Diagram 33- Closest packing of circles

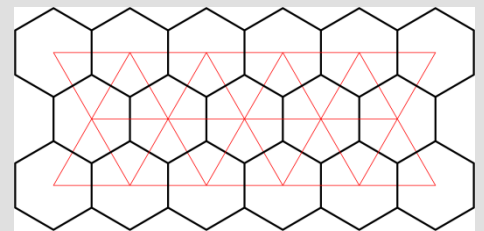


Diagram 34 - Closest packing of hexagons

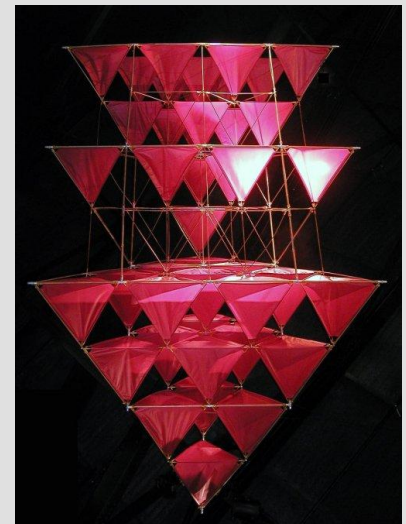


Figure 25 -A model of a Bell kite on display at the Cradle of Aviation Museum in Garden City N.Y.

⁴² Structure in Nature is a Strategy for Design – Peter Pearce.

Polyhedron

A polyhedron is a volume bounded by plane surfaces. It may be considered as finite set of connected plane polygons forming a closed volumetric figure, such that each side of every polygon is shared with one other polygon, and that the sides of these polygons intersect at their ends in groups of three or more to form the vertices of the polyhedron. The polygons are the faces and their sides the edges of a polyhedron. A polyhedron is regular if it's faces are identical (congruent) regular polygons and equal, and all of its vertices are equivalent, i. e., all its vertices are surrounded alike.

The cube is the only platonic polyhedron that will repeat to fit all space. It is the most symmetrical variation on the infinite class of three-dimensional figures known as parallelepipeds. The parallelepipeds are prisms whose bases and sides are parallelograms, they are, and therefore six faced polyhedron. Any parallelepiped will fill space by the congruent repetition of itself.

Close packing of spheres

In geometry, close-packing of spheres is a dense arrangement of equal spheres in an infinite, regular arrangement (or lattice). In a planar closest packing of spheres, each sphere is surrounded by six others it a second layer of spheres is added, the spheres are positioned vertically above the spaces between the spheres of the first layer.

The two ways to closest pack equal spheres are

- Hexagonal Closest Packing (HCP)

If the spheres in every third layer (of a repeating array of closest packed spheres) are positioned vertically above the spheres of the first layer, the resulting arrangement is called hexagonal closest packing. This is because the vertices of hexagonal prism can be defined by the positions of certain sphere centres in the array.

- Face Centred cubic close packing (FCC)

The spheres in every third layer are positioned vertically above the spaces of the first layer that are not covered by the second layer, the resulting arrangement is called cubic closest packing.

These are manifestations of two fundamental symmetry classes in the theory of periodic structures. In both systems, the spheres are arranged in equilateral triangular order and each sphere is surrounded alike by twelve others.

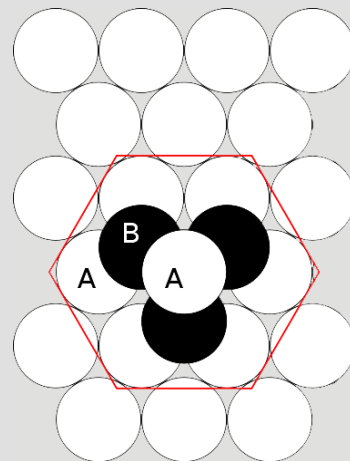


Figure 26 – Hexagonal Closed Packing.

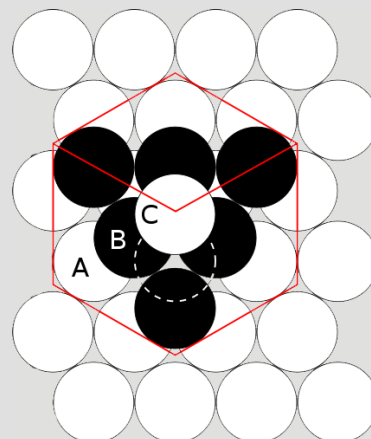


Figure 27 – Face Centred Cubic Closed Packing.

Voronoi tessellations of regular lattices.

Voronoi tessellations of regular lattices of points in two or three dimensions give rise to many familiar tessellations.

- A 2D lattice gives an irregular honeycomb tessellation, with equal hexagons with point symmetry; in the case of a regular triangular lattice it is regular; in the case of a rectangular lattice the hexagons reduce to rectangles in rows and columns; a square lattice gives the regular tessellation of squares; note that the rectangles and the squares can also be generated by other lattices (for example the lattice defined by the vectors $(1,0)$ and $(1/2,1/2)$ gives squares)
- A 3D cubic lattice gives the **cubic honeycomb**

The cubic honeycomb is the only regular space-filling tessellation (or honeycomb) in Euclidean 3-space, made up of cubic cells. It has 4 cubes around every edge, and 8 cubes around each vertex

- Parallel planes with regular triangular lattices aligned with each other's' centres give the **hexagonal prismatic honeycomb**.

The hexagonal prismatic honeycomb is a space-filling tessellation (or honeycomb) in Euclidean 3-space made up of hexagonal prisms.

It is constructed from a hexagonal tiling extruded into prisms.

It is one of 28 convex uniform honeycombs.

- Certain body centered tetragonal lattices give a tessellation of space with **rhombo-hexagonal dodecahedra**.

The **rhombo-hexagonal dodecahedron** is a convex polyhedron with 8 rhombic and 4 equilateral hexagonal faces.

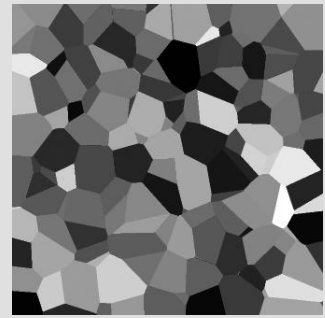


Figure 28 -This is a slice of the Voronoi diagram of a random set of points in a 3D box. In general a cross section of a 3D Voronoi tessellation is not a 2D Voronoi tessellation itself.

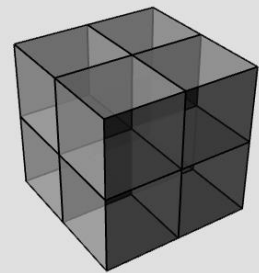


Diagram 35 -Cubic Honeycomb

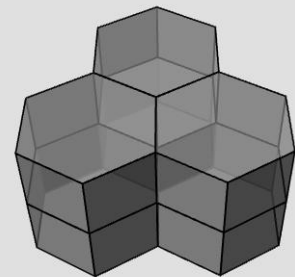


Diagram 36 –hexagonal prismatic honeycomb tiling

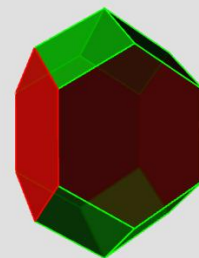


Diagram 37 –
rhombo
hexagonal
dodecahedron

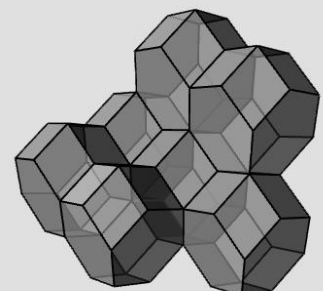


Diagram 37.a –rhombo hexagonal dodecahedron tiling

- A body-centred cubic lattice gives a tessellation of space with truncated octahedra.

In geometry, the **truncated octahedron** is an Archimedean solid. It has 14 faces (8 regular hexagonal and 6 square), 36 edges, and 24 vertices.

- A face-centred cubic lattice gives a tessellation of space with **rhombic dodecahedra**.

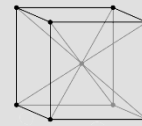


Fig –body centric cubic lattice

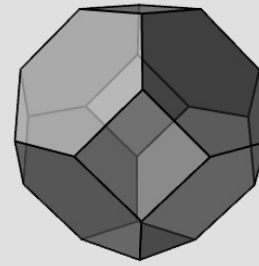


Diagram 38 –truncated octahedron

Filling the void.

Just as the closest packed circles left concave spaces between them, so does an array of closest packed spheres. As we might suspect, these concave shapes have high surface area per unit volume and consequently depreciate the minimal surface advantage of the spheres. If the spheres are swollen to fill in these concave voids, a polyhedron results with 12 identical rhombic faces. It is known as the rhombic dodecahedron.⁴³

Rhombic Dodecahedron

Rhombic Dodecahedron is a convex polyhedron with 12 rhombic faces.

It has exactly 12 faces as it is formed from the array of closest packed sphere which is alike surrounded by 12 others.

A dual network for space filling polyhedral may be found by constructing edges which join the centres of every polyhedron in the space filling array with the centres of its neighbouring polyhedral which share common faces. So the network which is dual to space filling array of rhombic dodecahedron is the space filling array of tetrahedra and octahedra in which 12 edges meet at the vertices which fall at the centres of the original rhombic dodecahedra.

Rhombic dodecahedra honeycomb is the Voronoi diagram of the face-centred cubic sphere – packing which is believed to be the densest possible packing of equal spheres in ordinary space.

Similarly the 3d Voronoi diagram for spheres arranged in hexagonal closest packing is formed by trapezo-rhombic dodecahedron in a special case.

Trapezo-Rhombic Dodecahedron

The trapezo-rhombic dodecahedron is a convex polyhedron with 6 rhombic and 6 trapezoidal faces.

This shape could be constructed by taking a tall uniform hexagonal prism, and making 3 angled cuts on the top and bottom. The trapezoids represent what remains of the original prism sides and the 6 rhombi a result of the top and bottom cuts.

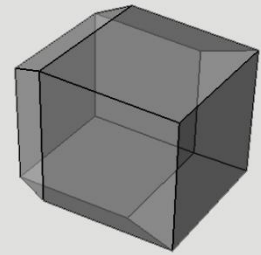


Diagram 39 –Rhombic dodecahedron

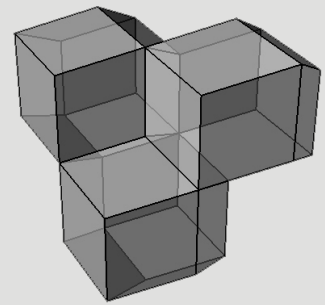


Diagram 39.a –Rhombic dodecahedron tiling

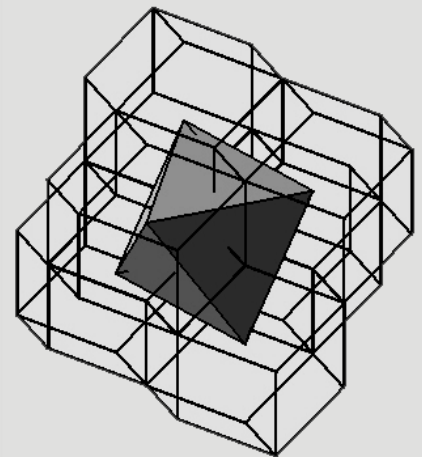


Diagram 39.b –A Dual Network of Rhombic dodecahedron

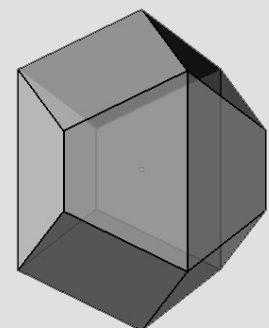


Diagram 40 – Trapezo – Rhombic Dodecahedron.

⁴³ Structure in Nature is a Strategy for Design – Peter Pearce.

Thus close packing in 3d of regular points results in 3d Voronoi.
But when such polyhedrons are understood with the aspect of being most efficient, it makes us inquire the variable of volume/surface area.

What is the most efficient way to divide space into cells of equal size with the least surface area between them?

- Lord Kelvin

Describing Tetrakaidecahedron and Weaire-Phelan Structure.

Tetrakaidecahedron.

"Space filling " means the combining of like or complimentary bodies

If a three-dimensional packing is continuously repeated, in such a way that there is no unoccupied space, it is known as "space-filling".

From the understanding of rhombic dodecahedron, can it be considered as the shape with least surface area per unit of volume that will uniformly partition space?

Lord Kelvin himself proposed a highly regular solution to his own problem.

14-sided Kelvin shape (called a tetrakaidecahedron)

Lord Kelvin (1887) showed that there is one shape made up of plane surfaces that will uniformly partition space with less surface area than rhombic dodecahedron, This shape is called truncated octahedron, a polyhedron bounded by six square faces and eight regular hexagonal faces, a total of 14 faces. The standard plane faced version of this shape has approximately 1 percent less surface area for a given volume than the rhombic dodecahedron. Kelvin also proposed a "minimal tetrakaidecahedron" in which a slight saddle curvature is given to the hexagon faces of the truncated octahedron; the result is approximately 0.103 percent less surface for a given volume than for the truncated octahedron.⁴⁴

Kelvin's solution, the tetrakaidekahedron remained the best solution to this problem until 1993 when Denis Weaire and Robert Phelan found a structure with 0.3% less surface area than Kelvin's structure.

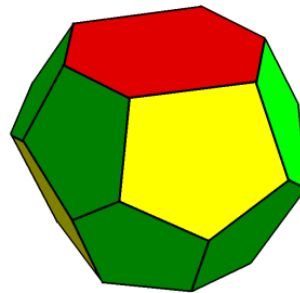


Diagram 41 – Tetrakaidecahedron.

⁴⁴ Structure in Nature is a Strategy for Design – Peter Pearce.

Weaire-Phelan Structure.

In geometry the Weaire-Phelan structure is a complex three-dimensional structure discovered by Professor Denis Weaire and Robert Phelan, two physicists based at Trinity College Dublin

The Weaire-Phelan structure differs from Kelvin's in that it uses two kinds of cells, though they both have equal volume.

One is an irregular dodecahedron with pentagonal faces, possessing tetrahedral symmetry

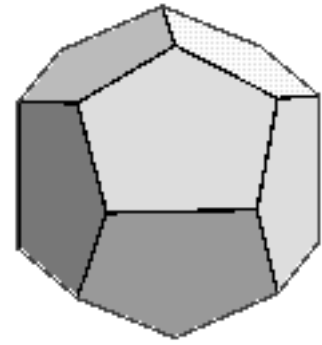


Diagram 42 – Dodecahedron.

The second is a tetrakaidecahedron with two hexagonal and twelve pentagonal faces possessing antiprismatic symmetry (D_{2d}).

Like the hexagons in the Kelvin structure, the pentagons in both types of cells are slightly curved.

The surface area of the Weaire-Phelan structure is 0.3% less than that of the Kelvin structure. It has not been proved that the Weaire-Phelan structure is optimal, but as of 2010 no better structure has been reported.

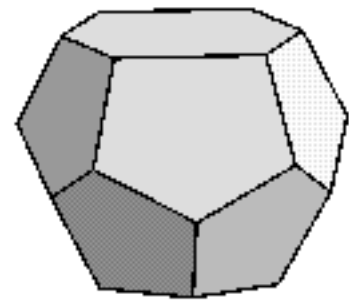


Diagram 43 – Tetrakaidecahedron

In the packing, two irregular pentagonal dodecahedra (12-sided) and six tetrakaidecahedra (14-sided) form a translation unit with a lattice periodicity which is simple cubic.

The illustration shows the dodecahedra as wire frames, and the tetrakaidecahedra as solid. The dodecahedra do not touch each other, but are entirely surrounded by tetrakaidecahedra⁴⁵.

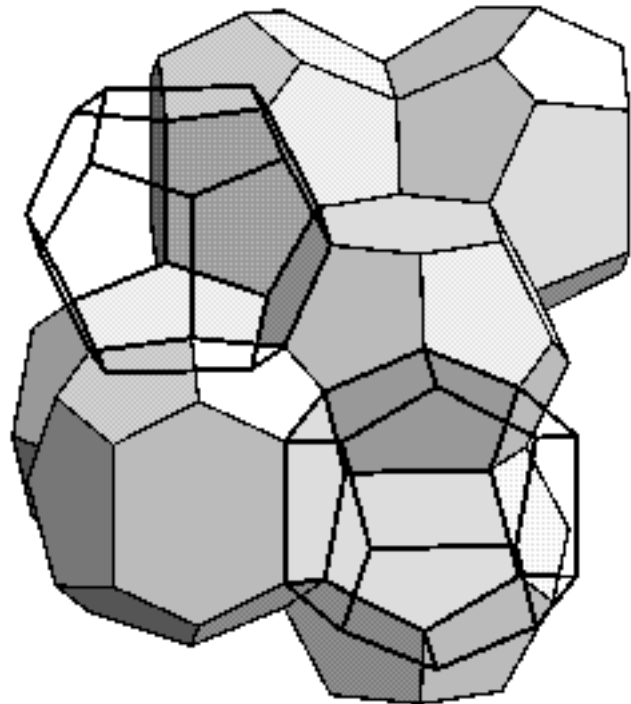


Diagram 44 – Weaire-Phelan Structure

⁴⁵ <http://www.steelpillow.com/polyhedra/wp/wp.htm>

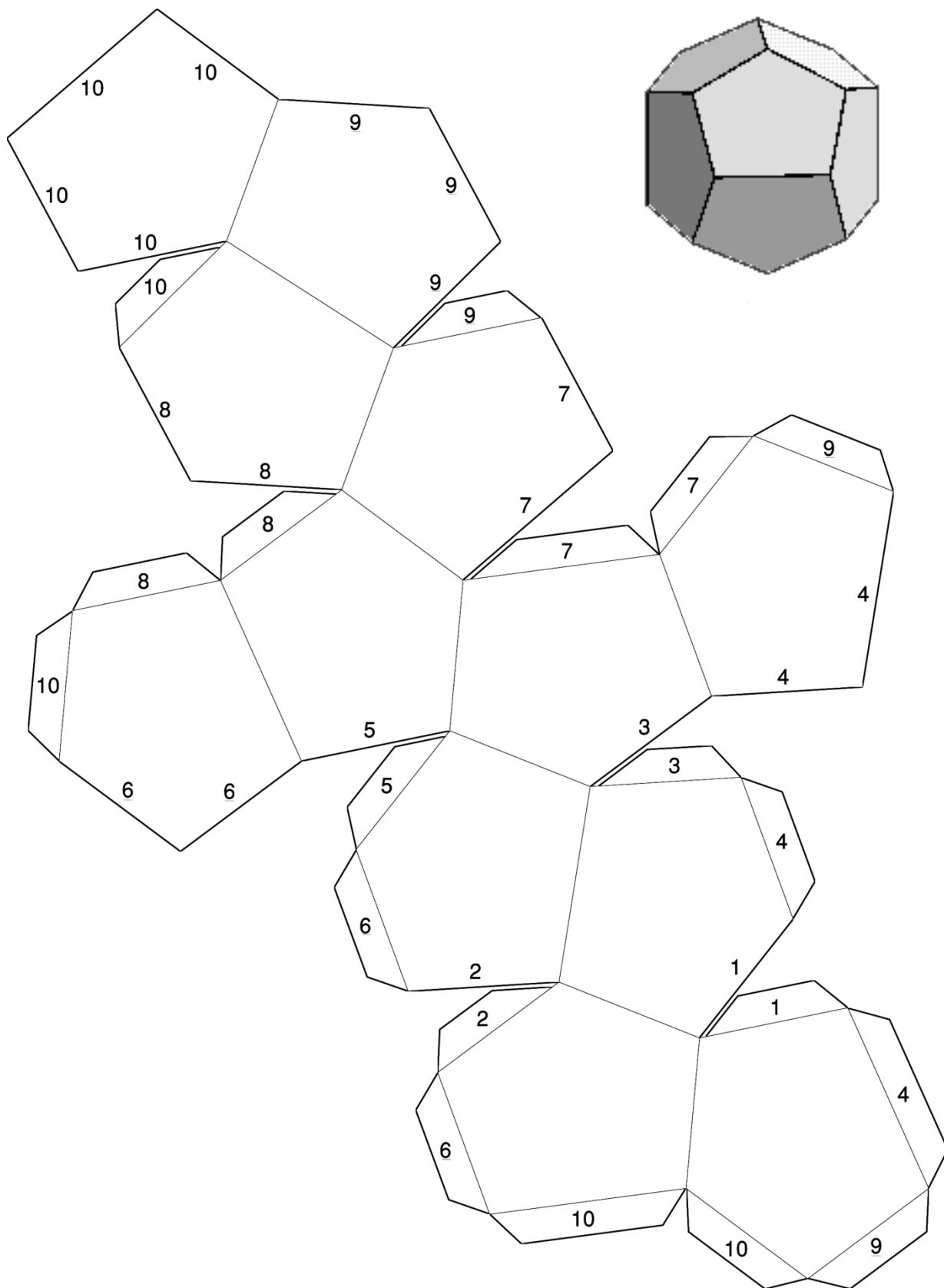


Diagram 45 – Cutting pattern for Dodecahedron.

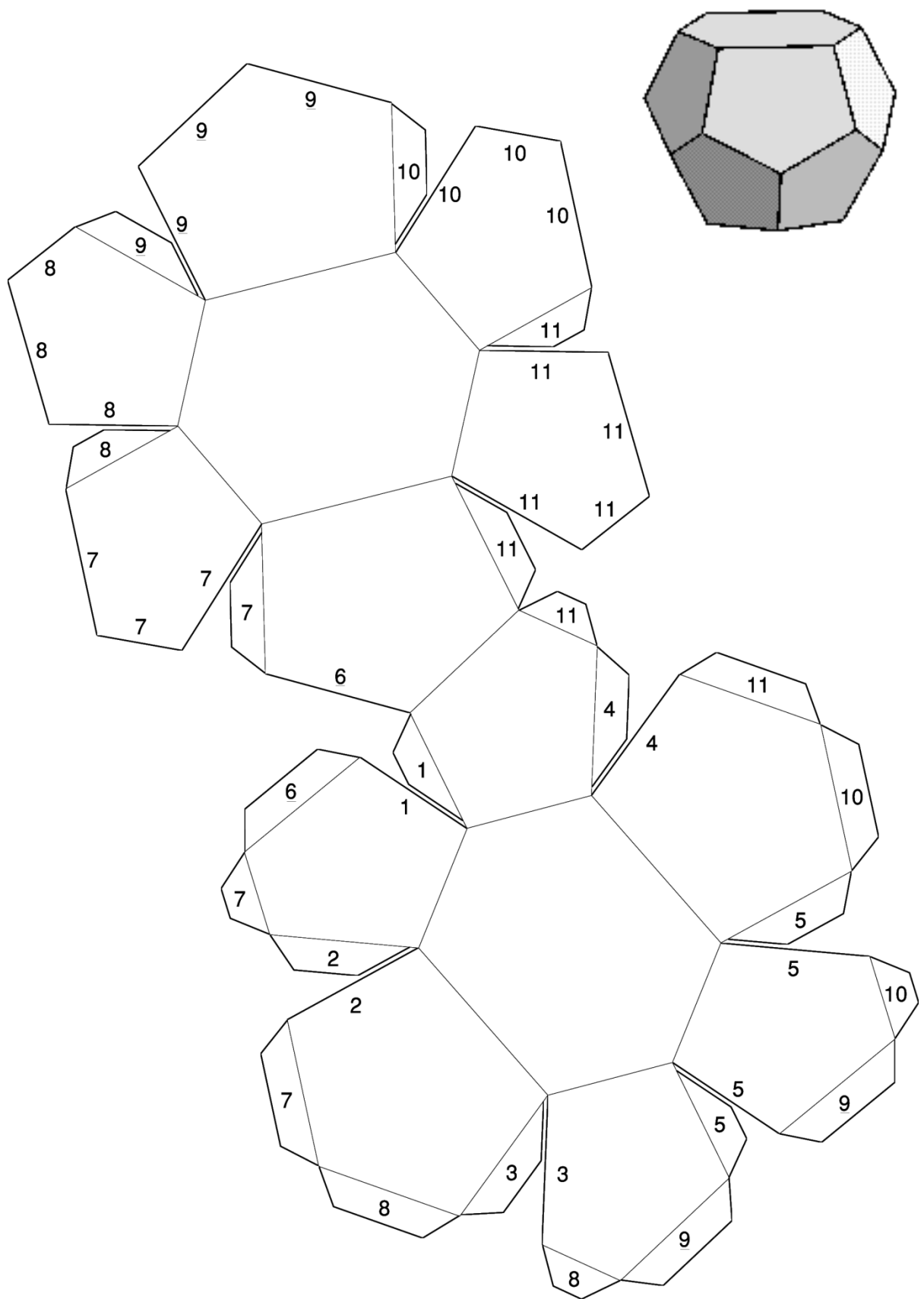


Diagram 46 – Cutting pattern for Tetrakaidecahedron.

As mentioned earlier all the case studies are explained under five categories, which are:

- Design Metaphor
- Strategy
- Material Behaviour
- Structural Performance
- Installation Methodology

The analytical framework thus deals with two aspects.

First of which is about the overall application of Voronoi algorithm in the respective projects. In this case too all the categories are interrelated, but they are defined separately for a better understanding of the algorithm.

Second aspect is based on the analysis of looking at application of Voronoi algorithm based on an approach, i.e.

- Network-based approach
- Domain-based approach
- Boundary-based approach

An example for the framework:

For any case study:

Chart explaining the first aspect

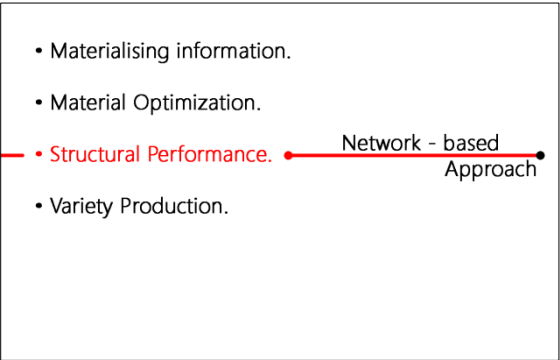
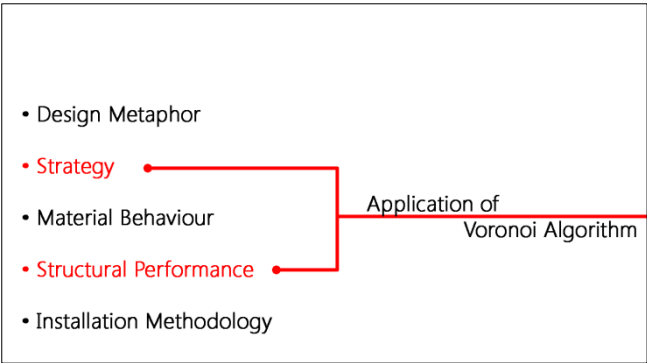
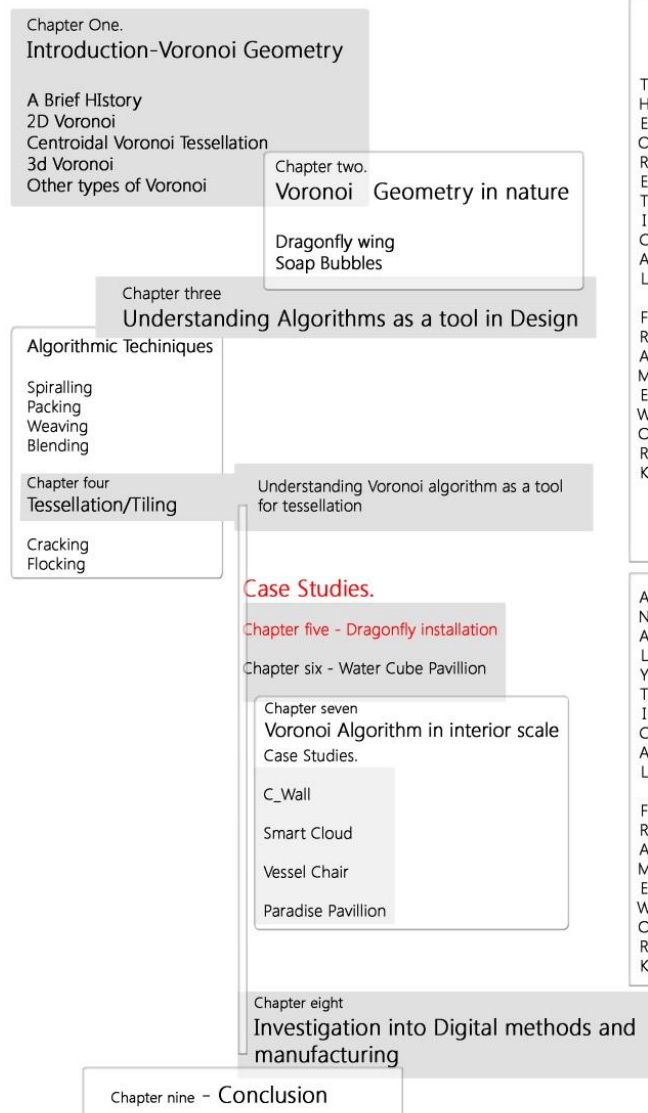


Chart explaining the second aspect



DRAGONFLY

Location

Southern
California Institute of Architecture (SCI-Arc)
Los Angeles, CA USA.

Services Performed by Buro Happold for the Project
Structural Design, 3D Parametric Fabrication Model
within CATIA environment, Assembly and Erection.

Client's Formal Name

SCI-Arc, Gallery Installation, 18 May 2007 to 8 July 2007.

Architect's Formal Name

Tom Wiscombe, founder of Los Angeles based design
firm EMERGENT.

*"In nature the dragonfly wing is unmatched in its structural performance and exquisite formal variation. Its morphology cannot be traced to any single bio-mathematical minima or optimum, but is rather the complex result of multiple patterning systems interweaving in response to various force flows and material properties"*⁴⁶

Tom Wiscombe

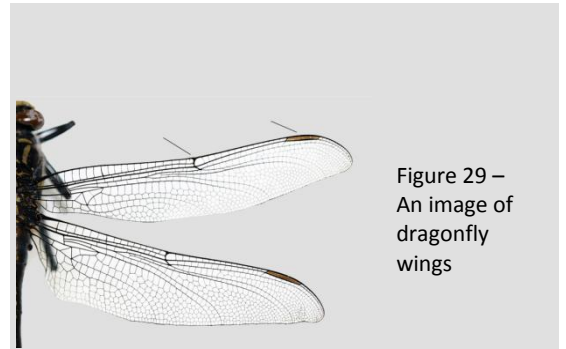


Figure 29 –
An image of
dragonfly
wings

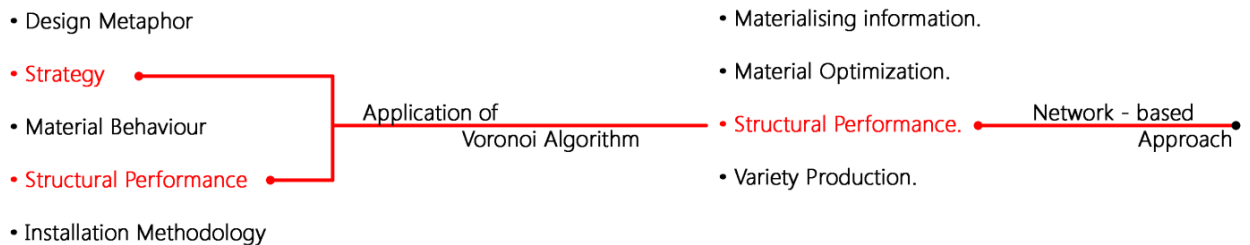


Figure 30 – Dragonfly installation



Figure 31 – Workshop for Dragonfly installation

Analysis chart



⁴⁶ <http://www.emergentarchitecture.com/>

By its name it suggests that here Voronoi algorithm is used as a design metaphor, but the overall form do not reflect its implication directly, thus in the analysis chart it has not been mentioned as a criteria.

Design Metaphor.

Inspired by the cellular morphology of the wing of an insect, dragonfly is a highly irregular grid-shell that cantilevers 35ft (10m) from its supports.

It investigates the extreme structural and formal properties of Dragonfly wing.

Dragonfly wings consist of both honeycomb patterns which are flexible and exhibit membrane behaviour and ladder-type patterns which are stiff and exhibit beam-like behaviour. These patterns are characterized by their rule-based interaction in terms of cell density, cell shape, and cell depth, as well as other parameters affecting overall wing performance, such as out-of-plane pleating behaviour and material distribution. A composite of distributed and linear structural formations, the dragonfly wings are fields of continuous variation and adaptation evolving toward overall robustness.

In this installation, dragonfly morphology and syntax are employed biomimetically rather than biomorphically, that is in terms of formal and behavioural logics rather than pure aesthetics.⁴⁷



Figure 32 – A detailed image showing Dragonfly installation



Figure 33 – Dragonfly installation before the exhibition

⁴⁷ <http://europaconcorsi.com/projects/17413-Dragonfly>

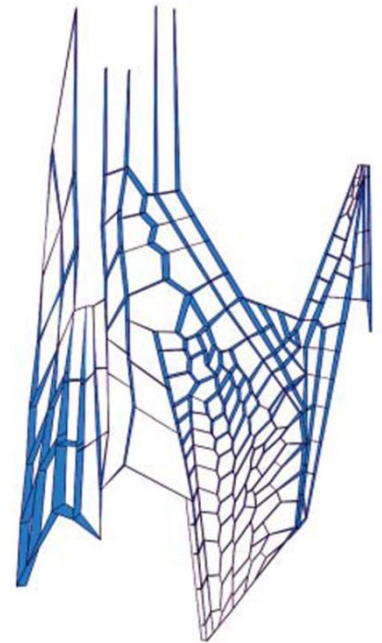
Behaviours

Veins – Emergence of quad cells and accumulation of material in order to create stiffness.

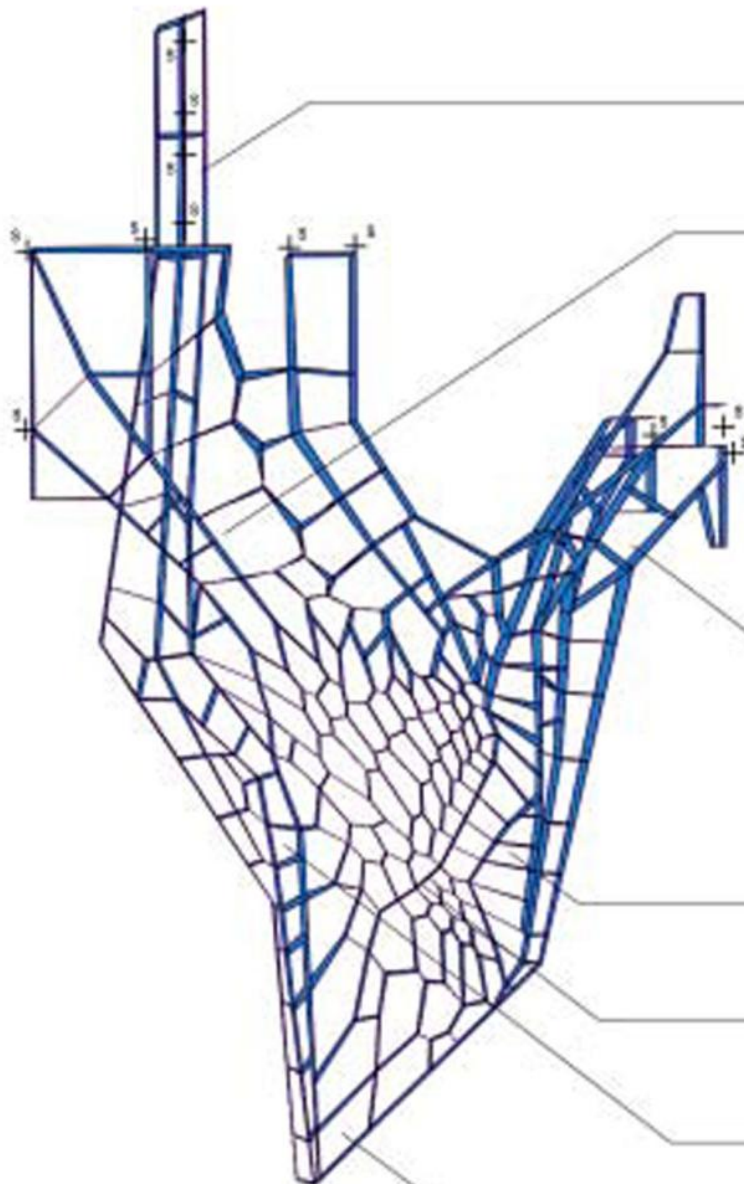
Pleats – Folding of the surface creates localized depth which together with material accumulation and cell-shape, creates beam – action.

Curvature- Surface curvature particularly in multi-sided cells creates membrane-action. Curvature of veins allows for hybrid interlacing of various patterning.

- Cell size and density controlled by boundary condition
- Veins take on complex shapes according to force flow.
- Honeycomb hybridizes with beams into composite system.
- Gradient more continuous yet highly differentiated.



- Extreme cell size and density differences.
- Veins tend to be linear.
- Honeycomb occurs primarily as infill.
- Discontinuities evident in gradient.



Veins align with existing steel of mezzanine and organize into quad cells for stiffness.

Veins extend out of the honeycomb and connect to existing catwalk for stability

Arm determinates and pleats to connect to existing column and create beam action.

Vein splits and hybridizes with honeycomb in response to indeterminate condition.

Vein emerge to create continuity through honeycomb

Vein emerges and pleats consolidating large loose cells into stiff beam

Cells at end of cantilever begin to thin out in order to reduce material weight

Strategy.

The form and the patterning of tessellations were developed through an adaptive approach that looked for emergent force path and deformation characteristics.

Application of Voronoi Algorithm

The criterion was to design a structure and to measure its performance with the maximum length of cantilever as a variable.

Type of Voronoi Algorithm used: Centroidal Voronoi tessellation.

These honeycomb pattern (Centroidal Voronoi Algorithm) are flexible and exhibit membrane behaviour, and ladder-type patterns, which are stiff and exhibit beam like behaviour.

To achieve the cantilevered condition, digital optimization routines were employed to refine the structure, as well as to create formal variation in response to local conditions.

This honeycomb pattern was then morphed to exhibit a hybrid configuration

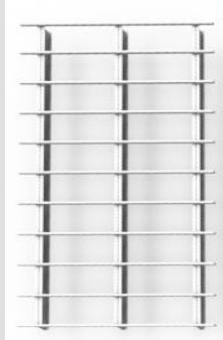


Figure 34.a -

Trabeated

- Hierarchical.
- Performance associated with primaries.
- Secondaries as infill only.
- Linear force flow.

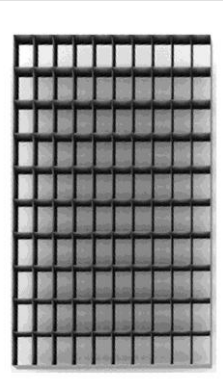


Figure 34.b -

2-Way Plate.

- Non -Hierarchical.
- No response to local conditions.
- Good stiffness.
- Unresponsive to indeterminate force flow.

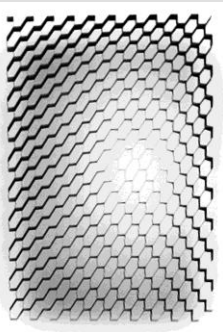


Figure 34.c -

Honeycomb plate.

- Non -Hierarchical.
- No in-plane stiffness
- Flexible infill.
- Unresponsive to indeterminate force flow.

Figure 34.d -

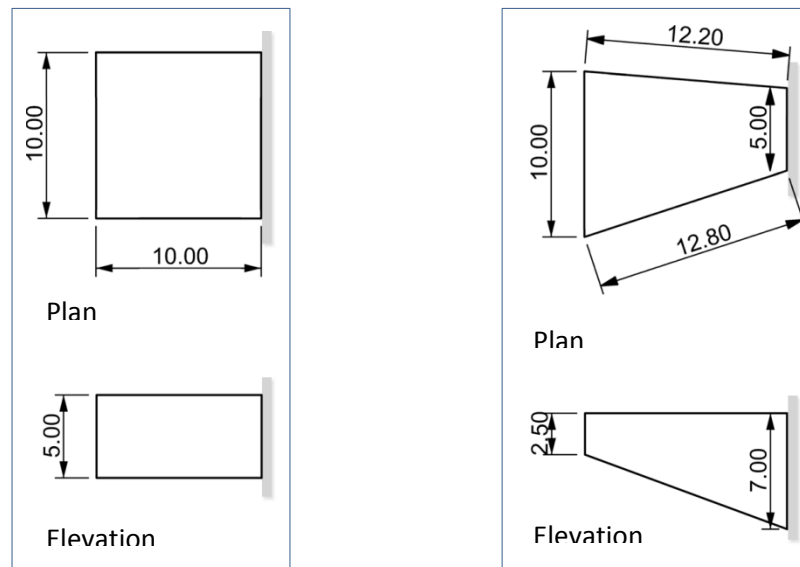
Dragonfly Composite.

- Emerging structural hierarchy in distributed field.
- Localized in-plane stiffness (quad cells).
- Localized flexible infill (honeycomb cells).
- Adaptive response to indeterminate force flow.

To control vertical deflection, the depth of the panels or the curvature of the shell might be adjusted. On the other hand, in-plane instabilities resulting from shear deformation were controlled by reconditioning or adjusting the geometry to align with the dominant force paths. In this way, factors such as the density of mesh, cell type (i.e. four, five or six-sided polygons), thickness of cell wall and overall surface topology were adjusted through a performance driven optimization

Now if seen, diagrammatically, the Centroidal Voronoi configuration is morphed and the imbalance in the structure is balanced by making changes in Z – dimension.

For example -



Dragonfly installation is an experiment on the fluid feedback of design sensibility, engineering innovation, and fabrication logic in a contemporary digital environment where these disciplines have become enmeshed like never before.

Dragonfly is governed by a different set of parameters including gravity and seismic loads, specific support locations and quality of those supports, flat material increments, and buckling failure, differences which lead to an unpredictable hybrid morphology. Seen in a larger context, this project contributes to the recent contemporary discourse on cellularity in architecture as a departure from pure cellularity toward a tectonic based on emerging structural hierarchies within rhythmic cellular fields.

Structural Performance.

Dragonfly deals with lateral connectivity, employing the depth of the bands to span. Using boundary conditions relating to overall structural shape, individual cell morphology (domain), vein distribution and pleating, depth, and incremental material thickness, the geometry was evolved simultaneously toward performance and wild variation.⁴⁸

ANSYS, a structural optimization loop was used in the search for emergent characteristics that would improve performance and increase heterogeneity in the structure.

Although optimization of material and form were emphasized, DRAGONFLY was meant to achieve a kind of wild, dynamic, spatial architecture. The designers sought performance and wild variation. In doing so, Dragonfly reconsiders optimality in engineering, which is often about idealized problem solving, and attempts a messy evolutionary process closer to that seen in nature.

The criteria to measure the performance solely was based on to achieve maximum cantilever. All the related parameter works in order to achieve such performance. Material was selected accordingly, the cell morphology, thicknesses of the pleats and the density of network all were designed accordingly.

Based on the method of analysis done in this study, dragonfly installation cannot be categorised under a domain based or boundary based approach. As mentioned earlier structural performance has to be a network based approach.

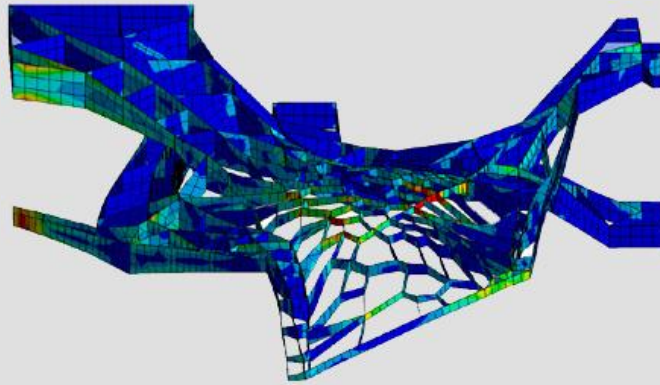


Figure 35-Structural Stress Analysis



Figure 36-Students installing the installation.

⁴⁸ Digital fabrication – Lisa Iwamoto

Material Behaviour.

Aluminium plate metal was the material of choice because it is light weight and easy to work with, and because it was found to be more cost effective to fabricate than steel. The assembly consisted of 455 individual bent plates that were threaded, or spliced, together in such a way that no welding was required. If indicated by analysis, additional plates could be “stacked” together to increase material where high stresses occurred. Each plate was cut from bulk 4ft x 8ft x 1/8in thick aluminium sheets using CNC technology.

Buro Happold developed a fabrication model, built in CATIA, that accounted for all plates, bend locations, bend angles and bolt hole locations. The model was built parametrically such that adjustments to the geometry could be quickly implemented and the bent plates could be “unfolded” for fabrication. Once unfolded, the plates were placed onto 4’x8’ sheets using an optimization/packing algorithm to minimize waste. The fabrication procedures for Dragonfly reflect an adaptive model. A CATIA fabrication model was generated which parametrically linked hundreds of two-dimensional unfolded bands to ‘live’ three-dimensional geometry⁴⁹

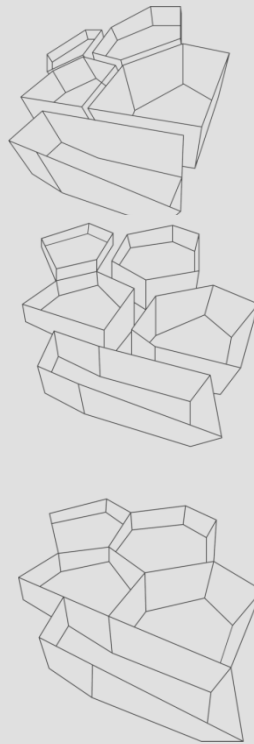


Figure 37.a -Digital model of test piece for mock up.



Figure 38 –Final mock up.

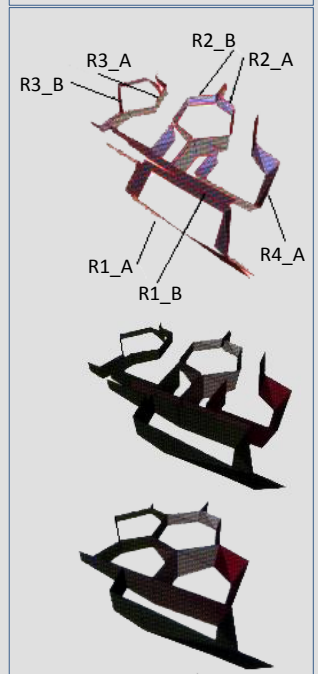
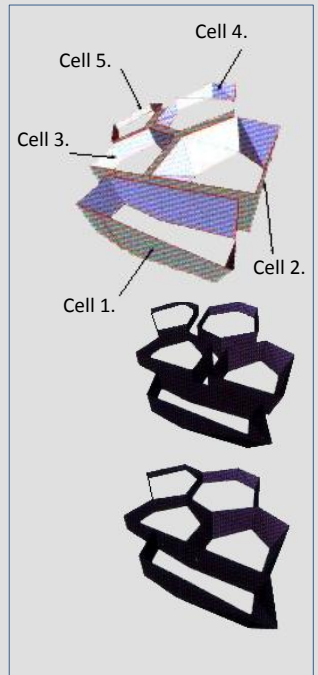


Figure 37.b -Digital model of test piece for mock up.

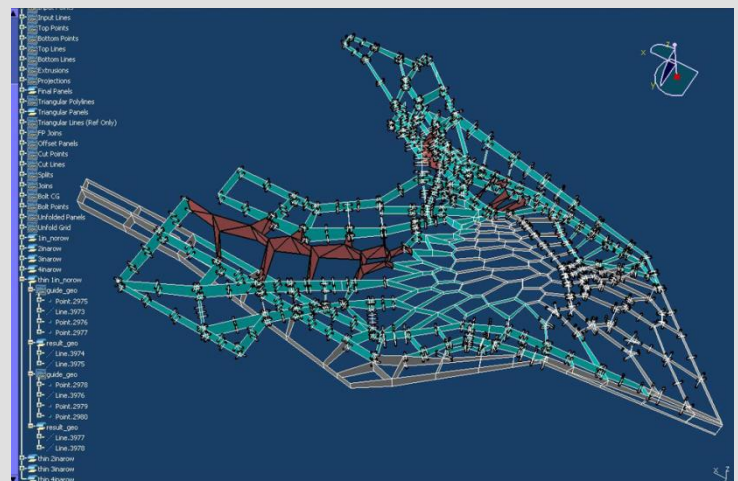


Figure 39 –CATIA fabrication model.

⁴⁹ Digital Fabrications- Lisa Iwamoto

Installation Methodology.(text)

As the design evolved and as engineering information was filtered into the fabrication model, these bands, including scoring, bending, drilling, and location information, were updated automatically. Bands were distributed onto 4'x8' aluminium sheets automatically using nesting software which optimized material usage. These sheets were then cut and inscribed using CNC milling machines

Populations of random structural mutations were generated and fitness-tested based on the given support and loading conditions in a feedback loop involving multiple generations. Using boundary conditions relating to overall structural shape, individual cell morphology, and vein distribution and pleating, depth, and incremental material thickness, the geometry was evolved simultaneously toward performance and wild variation.

Because all of the information required for assembly of the structure was embedded into the bands, including relative cell position, the construction of the Dragonfly can be considered an aggressively bottom-up process and a relief from 'construction documentation' as it is currently understood by the architectural profession at large.⁵⁰

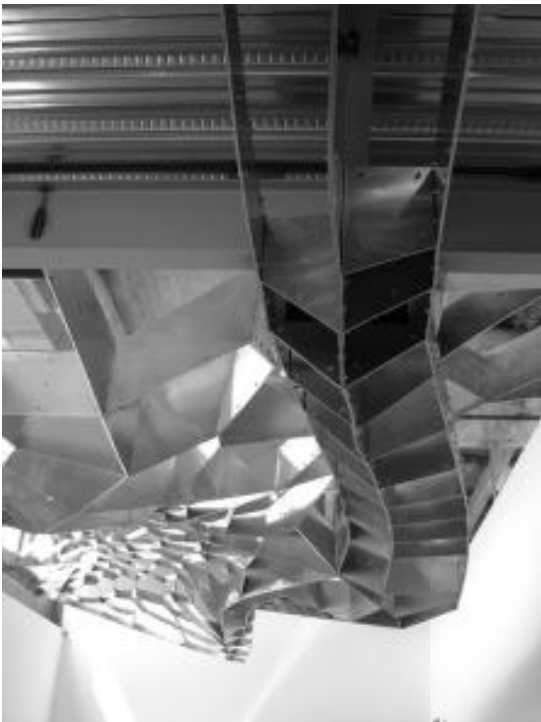


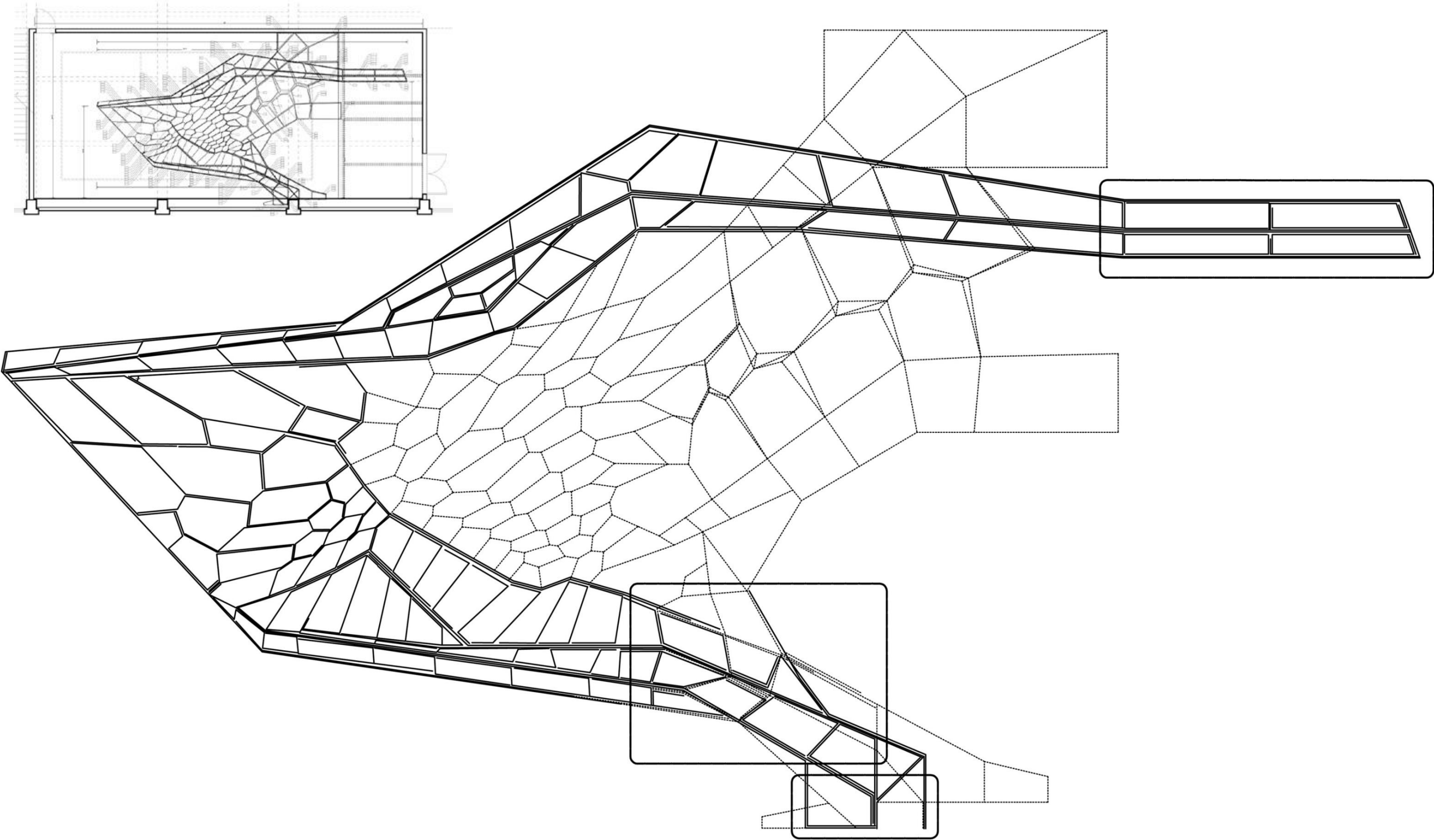
Figure 40, 41, 42, and 43 –Images of dragonfly installation

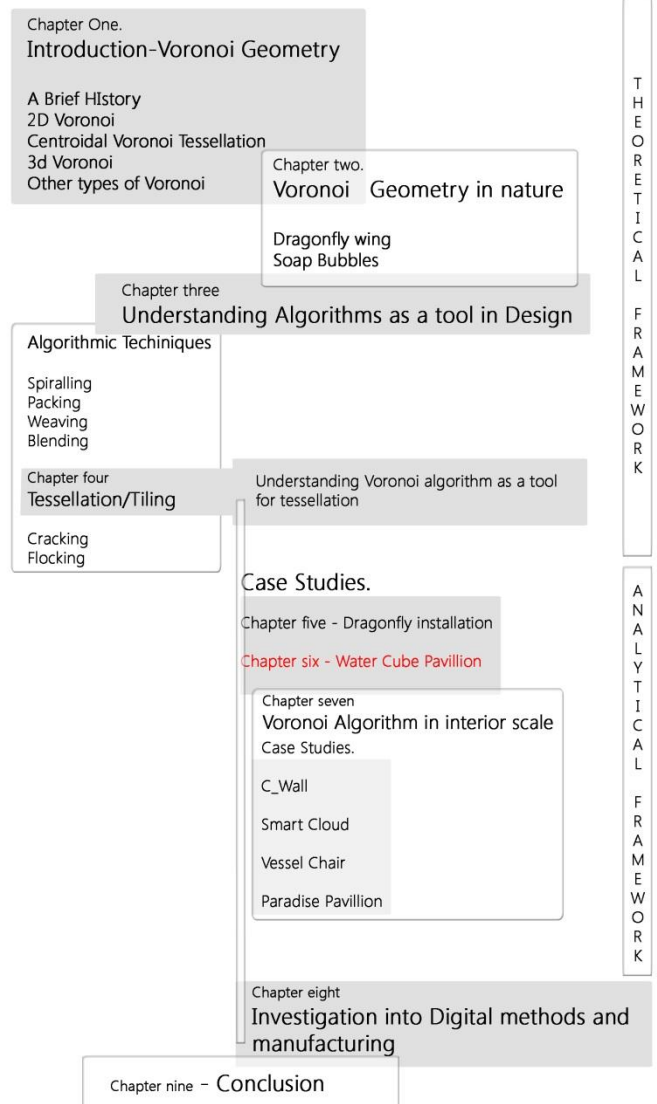
⁵⁰
http://www.sciarc.edu/sciarc_player.html?vid=http://www.sciarclive.com/Lectures/2010_02_12_MatthewMelnik.flv&title=Matthew%20Melnik

Based on the study of dragonfly installation, in this project Voronoi algorithm is applied in a strategic manner to achieve structural performance with the focused variable of cantilever.

- Type of Voronoi Algorithm used – Centroidal Voronoi Algorithm.
- Application – 2 dimensional.
- Maximum affected category
 - Strategy.
 - Structural Performance.
- Defined Approach based on analysis - Network based approach.

Drawing 1 – Plan of dragonfly installation.





Introduction

- Location
Beijing Olympic Green, Beijing, People's Republic of China
- Consultant
PTW Architects *In association with CCDI and Arup*
- Client
People's Government of Beijing Municipality, Beijing State-owned Assets Management Co., Ltd
- Date
2003

The water cube project for the Olympic Swimming Venue in Beijing is considered an architectural milestone in the field of computational design and construction and was highly anticipated as one of the most important buildings of 21st century.

The team on this project split into four groups to workshop design principles, cultural values, and environmental and structural concepts for the project. Early brainstorming of ideas quickly identified a range of possible seating bowl arrangements and developed design criteria for the form of envelope cover: the building should reflect the time and the place, be responsive to the immediate environment and its position as one of two gateway buildings; it should embody the concept of water in one of its forms; it should reflect ideas in traditional Chinese architecture and be culturally appropriate; and it should be a contemporary building – a cathedral for the Twenty-First Century – embodying the values and aspirations of China, testing materials, and expressing form in the most dynamic way.

For PTW, in particular, the team structure was crucial as it provided rich multi-cultural perspectives and diverse technological know-how that allowed for the technology transfer to take place. The end-product of the design process is the unique Water cube National Swimming Centre.

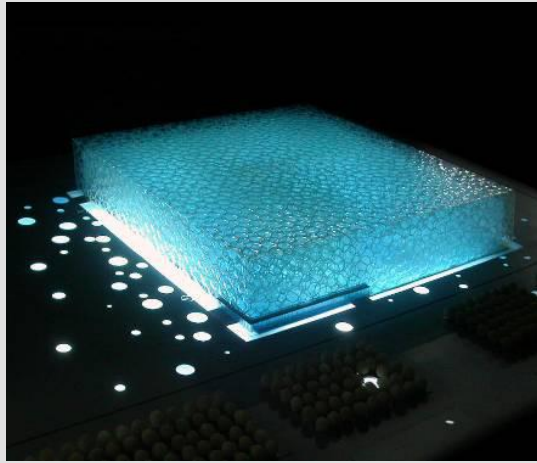


Figure 44 – Aerial View of the model of Water Cube Pavilion

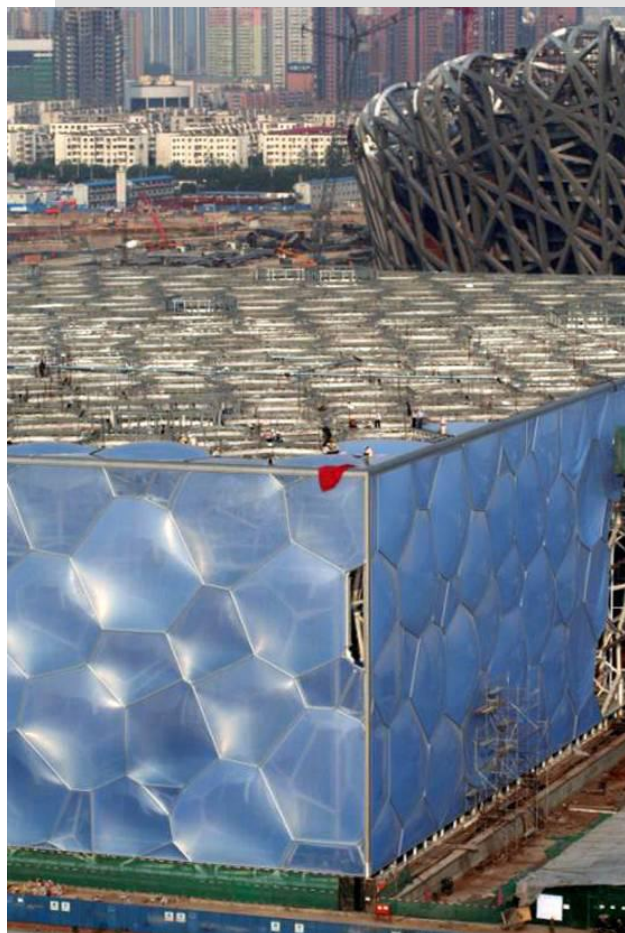


Figure 45 and 46 – Images of Water Cube Pavilion while under construction.

Urban Context

The National Swimming Centre is one of the two main venues for the 2008 Beijing Olympics. It sits directly across the main axis of the Olympic Green from the new National Stadium, nicknamed the 'Bird's Nest,' and designed by the Swiss firm Herzog & DeMeuron.

The Olympic Green Precinct lies at the northernmost end of Beijing's north-south axis, which originates at the central Forbidden City. The American planning firm of Sasaki Associates did master planning for the Green, which covers about 2,804 acres (1,135 hectares) and includes a nature preserve and Olympic Village, in addition to the two main stadia and eleven other venues. All of these elements have been designed with post-Games uses in mind and an eye toward sustainability.

The overall setting has a suburban scale, and lies on the northern fringe of the city. Surrounding blocks feature high rises on large building plots, and there is considerable open space within the Green itself.⁵¹

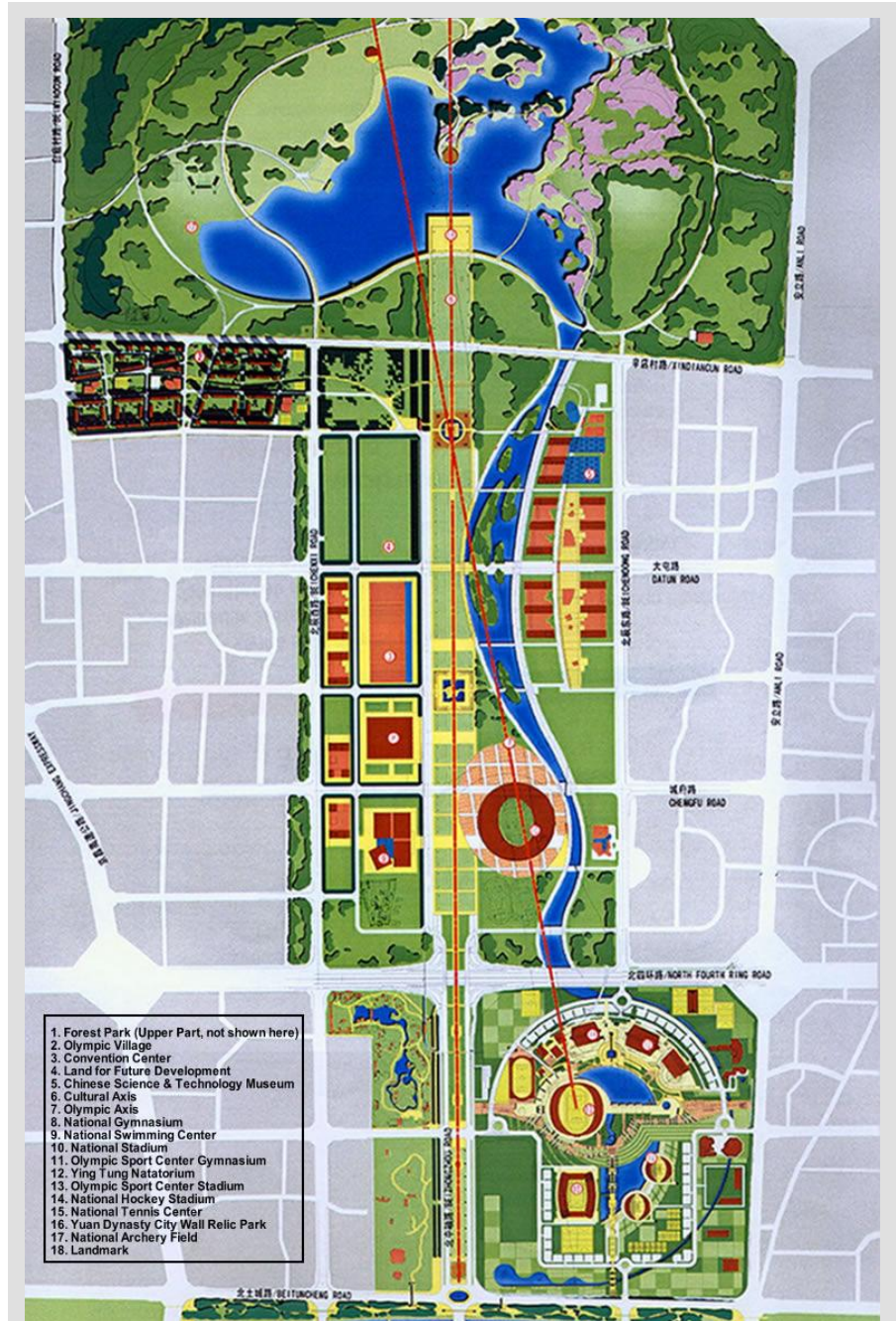


Figure 47 -Master Plan of Olympic Green.



Figure 48 -Aerial view of National Swimming Center and National Stadium under construction in 2005. Final roadways and plazas in the site are yet to be completed.

⁵¹ Miki Megumi, "Post-Olympic Age." *World Architecture News*.

http://www.worldarchitecturenews.com/index.php?fuseaction=wanappln.projectview&upload_id=366 (accessed 4 Dec. 2006)

In Water Cube Pavilion, as the name suggests the main criteria of the firm was that the building should reflect the notion of water. Thus Voronoi was used here with such criteria by the designers.

Conversation with the scientist of Weaire-Phelan structure.

Question – “In Water Cube pavilion-Beijing, the volume was defined by a notional cuboid and then after intersection with this geometry (Weaire-Phelan Structure) the internal structure was derived, but if the form of volume was anyways defined as a cuboid than how much appropriate is it to use this geometry for internal structure?”

Answer -

The minimal property in question is:

"(Infinite) space is to be divided into cells of equal volume with minimum interfacial area.

In the Water Cube, this perhaps irrelevant: the structure was chosen for aesthetic reasons!

I am not sure about your point concerning the external shape. In fact it would seem more logical to have wall perpendicular to the cubic axes of the periodic (crystal) structure. But, again, the designer saw that to "cut" walls at other angles would give a more disordered appearance, which is more aesthetic.

From an architectural point of view, what I find appealing is that the whole building has a completely coherent structure thought - no special interlocking or bracing at all in wall, ceilings, corners... I especially like the restaurant- brilliant!

It has sometimes seemed odd to me that a rectangular shape was used... but perhaps something more complicated would have taken things too far!

As it happens, we realised the aesthetic value of the structure long before this - but in a different sense. I'll send you a picture of a sculpture, originally conceived as a building (say, a large auditorium) - you might like to study this idea - it really is a very advantageous plan, not yet realised: it could surely be done on any scale.

The WP structure is close to but not quite a Voronoi construction. It has curved faces/edges. The designer used a Voronoi construction for convenience, and - again - "tweaked" it by varying the disposable parameters of the structure (the variations that preserve all symmetries) to optimise appearance.

The (brilliant) original designer was Tristram Carfrae - you would do to check him out for presentations, on the Web."

Denis Weaire
Emeritus Professor
TCD

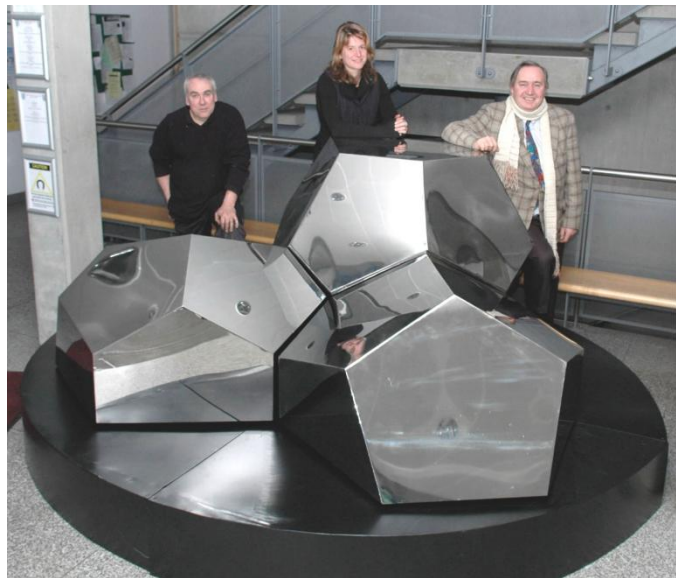


Figure 49 –A model of Weaire Phelan Structure.

Design Metaphor. (Text)

The Water cube associates as being a structural and thematically "leitmotiv" with the square, the primal shape of the house in Chinese tradition and mythology.

In combination with the main stadium by Herzog and de Meuron, a duality between fire and water, male and female, Yin and Yang is being created with all its associated tensions/attractions.

Also at this time Arup learned of the winning design for the nearby Olympic Stadium, the fantastic curvaceous red 'bird's nest' structure proposed by Herzog and de Meuron/Arup. With this inspiration, they quickly decided the swimming center should be a contrasting blue box – and so the 'Water Cube' was born.

Conceptually the square box and the interior spaces are carved out of an undefined cluster of foam bubbles, symbolizing a condition of nature that is transformed into a condition of culture. The appearance of the aquatic center is therefore a "cube of water molecules" - the water cube

Its entire structure is based on a unique light-weight construction, developed by PTW with ARUP, and derived from the structure of water in its aggregated state (foam).

Behind a seemingly random appearance hides a strict geometry, as can be found in natural systems such as crystals, cells and molecular structure - the most efficient subdivision of 3-dimensional space with equally sized cells.

The designers state, *"As a counterpoint to the exciting, energy-giving, masculine, totemic image of the National Stadium, the Water Cube appears as serene, emotion-engaging, ethereal and poetic, with changing moods that directly respond to people, events and changing seasons."*⁵²

The juxtaposition of the two forms contributes to an active dialogue between them.



Figure 50 – A rendered image of Water Cube Pavilion and Bird's Nest Structure.

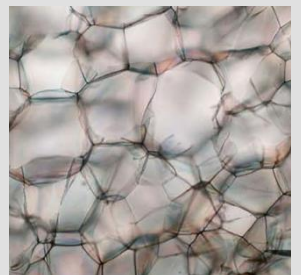
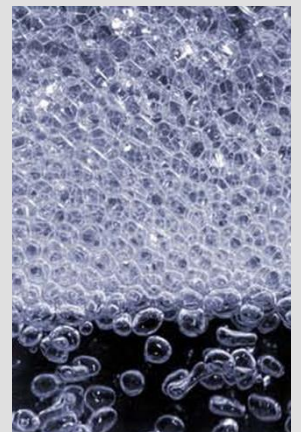


Figure 51 - These images display the intersection of structure and water, an intersection which drove innovation on this project.

PTW

Architects



⁵² Technical pages from UniPennReport_Watercube-1

Application of Voronoi Algorithm.

- Design Metaphor
- Strategy
- Material Optimization

Strategy and Design Metaphor

The prime challenge was to decide which form the structure should take and how the resulting cladding pattern would look. The consortium preferred the notion of a continuous skin that covered the walls and roof alike.

The team discussed a roof structure of vertical cylinders, clad top and bottom with circular panels. However, circles do not fit well together and there was the uncertainty of how to progress from vertical cylinders in the roof to horizontal cylinders in the walls without a clumsy intersection. So the question was generated that

"If cylinders do not work, what else will? What sorts of shapes fill three dimensional spaces uniformly, besides the somewhat prosaic triangulated space-frame?"

The answer was in the nature, from living cells to mineral crystals.

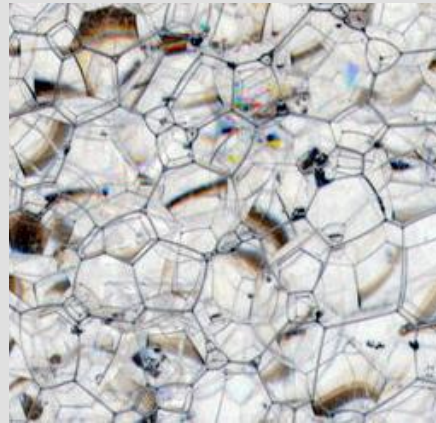


Figure 52 -Soap foam

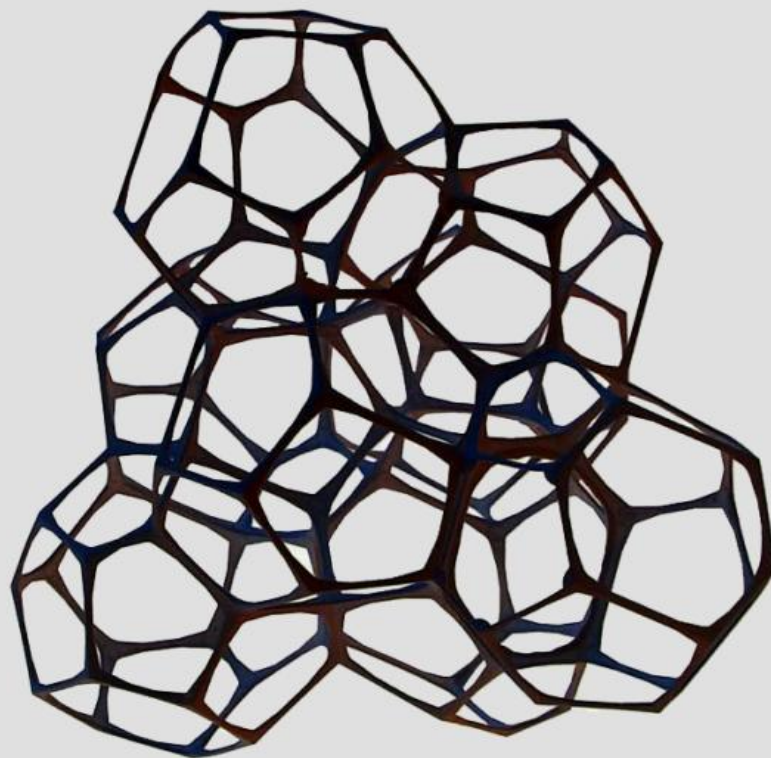
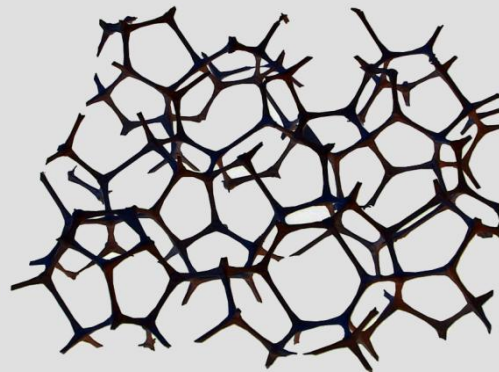


Figure 53 -Weaire – Phelan Structure

As swimming pools need to be heated for most of the year, it was decided that the solution that solved most of the technical problems was an insulated greenhouse.

Diffused natural light would enter through a main steel structure built within a cavity, to

isolate it from both the corrosive pool-hall atmosphere and from the outside. The architectural planning team concurrently calculated that the entire square site would be needed to fit all the required facilities into the center.

MODELLING THE IDEA

The search for the most efficient way of subdividing

space had led to a structure based on the geometry of an array of soap bubbles, clad with plastic pillows that look just like bubbles – to house the water at the heart of an aquatics center. The designer team decided to build an accurate physical model of all 22,000 steel tubes (which join at 12,000 nodes) and the 4,000 different cladding panels. Rapid prototyping machinery, more usually used by the manufacturing and automotive industries, seemed to offer the only hope, but no-one had ever made a model this complex. It took many weeks to learn enough about computer-aided design modeling and the data translation required just to make the structural model. With two days left, the model was flown from Melbourne to Beijing, where it was joined to a hand-made plastic skin – it had proved impossible to draw all the different pillow shapes in time. This completed model helped convince the judges and in July 2003 the consortium was announced the competition winner and awarded the design commission.

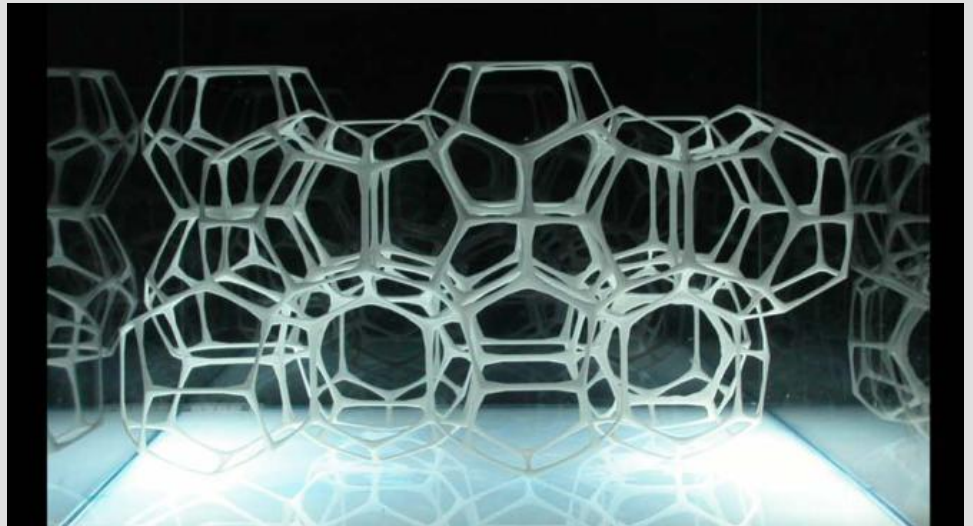
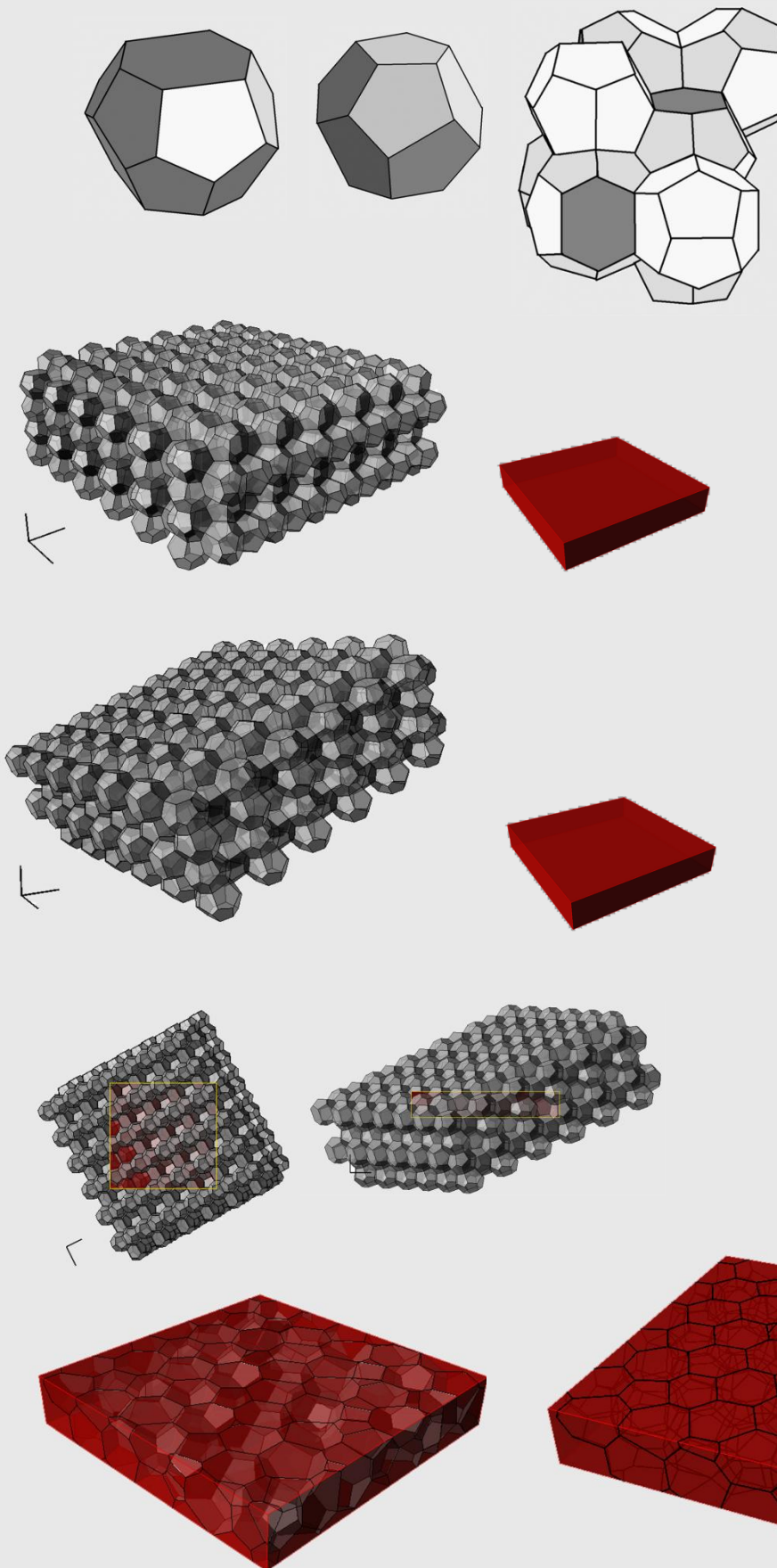


Figure 54 – Image of the model made by PTW Architects of Weaire – Phelan Structure



Figure 55 –A model of Weaire Phelan Structure.

Chart 4 – Application Of Weaire Phelan Structure in Water Cube Pavilion.



In spite of its complete regularity, when it is viewed at an arbitrary angle it appears totally random and organic.

Based on two 12-sided and six 14-sided polyhedrons, the cells are packed together in three-dimensional space to infinity. This repeating unit tiled in space is then rotated and cut along prescribed axes along the exterior to form the exterior geometry of a box, and then the interior cells are sliced to form the large interior spaces for the swimming facilities, etc. The polyhedron surfaces were then replaced with the building membrane and the edges became structural steel tubes. Where the steel members meet one another they are welded to large spherical steel nodes to form solid moment connections.

Structural Performance.(analytical models and drawings)

The Structure Facts:

- The wall cavity is 3.6mts deep and the cavity forming the roof is 7.2mts deep.
- The structure is made of approximately 6500 tonnes of steel.
- There are 22,000 steel members and 12,000 nodes
- The steel beams would stretch for 90 kms.
- The structure of the building is so strong that it can be stood up on its end and retains its shape.
- The overall size will be 177x177x31 mts.

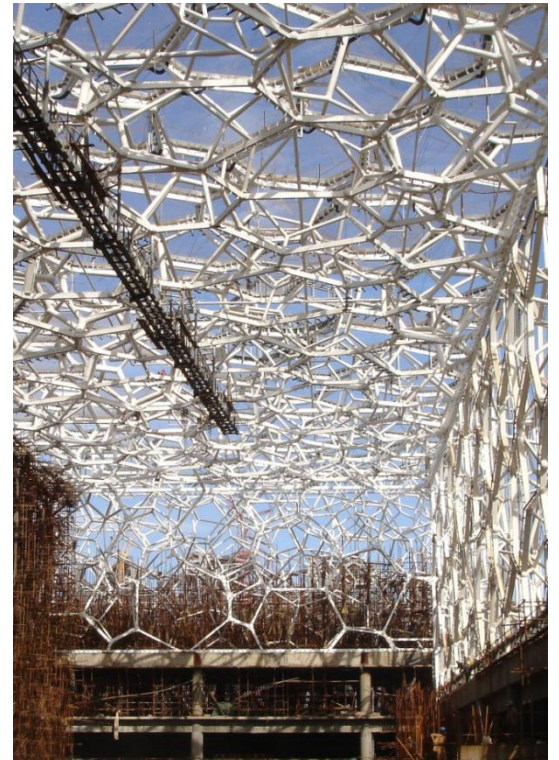


Figure 56 –A view of water cube pavilion from inside without the ETFE sheets. .

Technique and Technology

The computational powers of the computer were essential in the creation of the WaterCube geometry. The drawing system above allowed the engineers to draw, and redraw the members with complete accuracy. Because the structural optimization process was constantly evolving, the ability to update the geometry quickly was a top priority.

Structural Optimization Program

In order to perform the analysis of a structure as complex as the WaterCube, the engineers at Arup developed an optimization program that automated the process. Based on the designated design constraints and under over 190 different loading scenarios, the algorithm iteratively checked the distribution of forces through the entire structure based on specific member sizes. Based on the calculations the software would then determine the optimal size of a member and continue analyzing each additional member in the same way. Once complete, the algorithm measured the weight of each member and recorded the individual properties into a spread sheet. This process allowed the structural design team to test multiple designs based on a number of various variables regarding maximum and minimum diameters and weight for each of the 22,000 members in much less time and with far greater accuracy than if it had been done by hand.

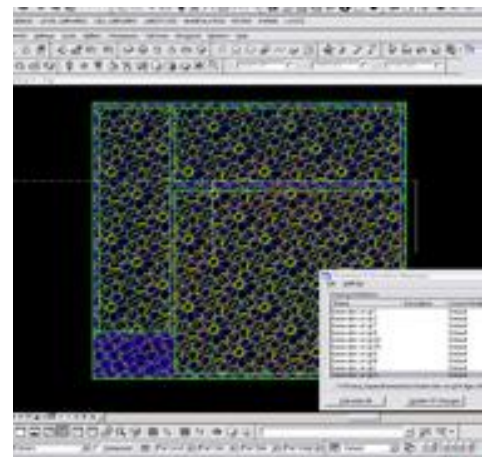


Figure 57 -Documentation extraction file

CAD Conversions

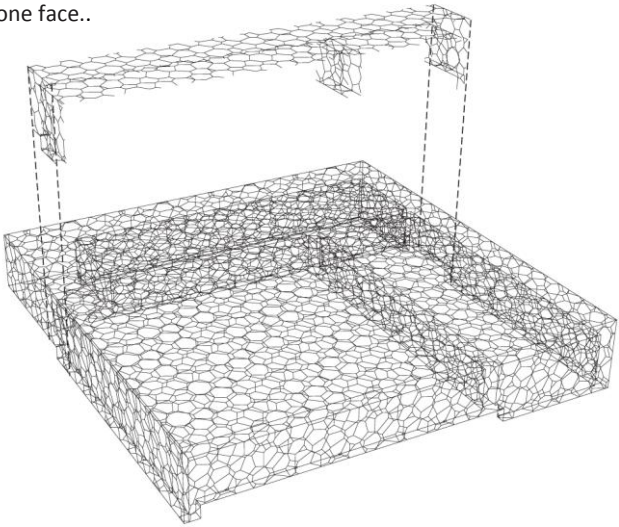
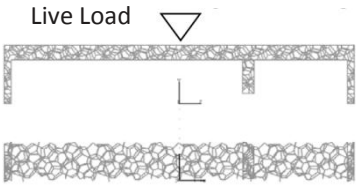
From the structural analysis model and the spreadsheet it produced, the engineers wrote a conversion program that ran inside of Microstation. This software took the dimensions of each line and generated a wireframe model of the entire WaterCube in twenty-five minutes, rather than the months that it may have taken by hand, with an accuracy only the computer can afford. This three-dimensional model was

further linked with other Bentley software from which Arup was able to extract all of the working drawings, including elevation, sections and details. These models were also passed on to the client and contractors who were then able to account for each and every building element.

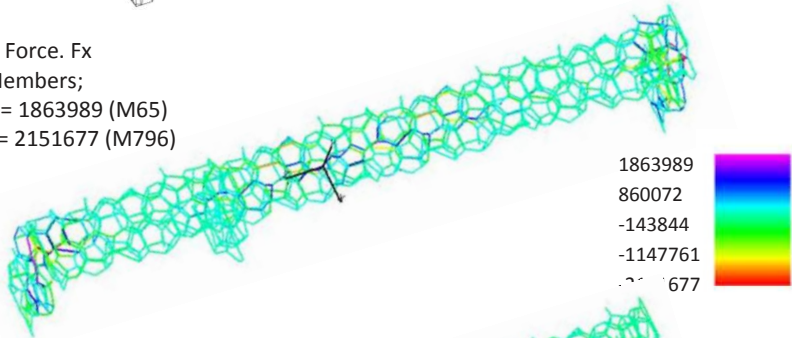
Bentley Structural

Inside Bentley Structural, the three-dimensional wireframe model was given its architectural properties. From the structural analyses material properties were integrated from the Bentley libraries of section sizes, even those members with odd dimensions where quickly and accurately modelled with precision. Bentley Structural automatically provided the engineers with a material list report contained all the

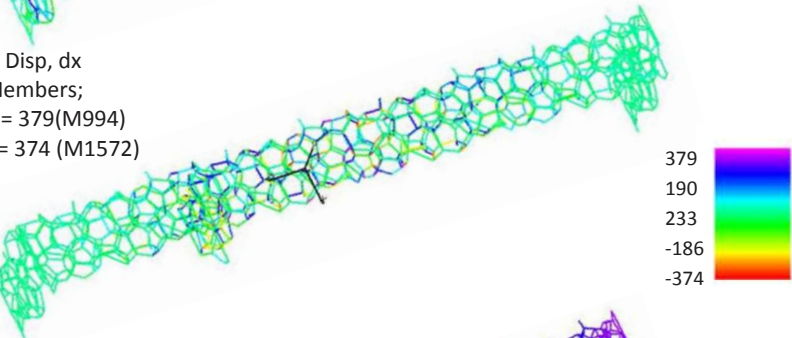
Using Visual Analysis™ software, the authors modeled a section of the WaterCube structure, applying material properties and end fixity conditions. This virtual model was then submitted to a variety of loads. Shown here are a mid-span vertical live loading condition and a direction wind load applied to one face..



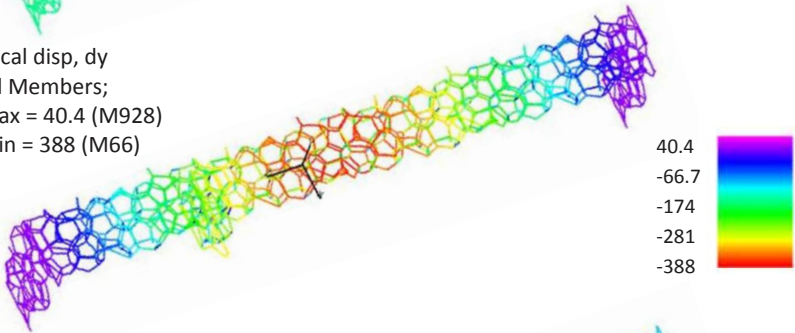
Axial Force. Fx
All Members;
Max = 1863989 (M65)
Min = 2151677 (M796)



Axial Disp. dx
All Members;
Max = 379(M994)
Min = 374 (M1572)



Local disp. dy
All Members;
Max = 40.4 (M928)
Min = 388 (M66)



Shear Force. Vy
All Members;
Max = 1408523 (M1260)
Min = 2365079(M1259)

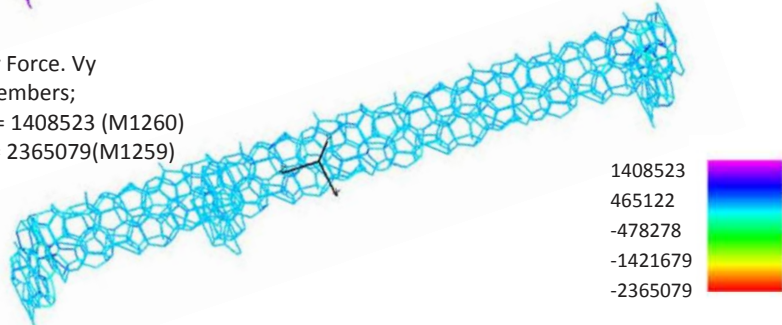
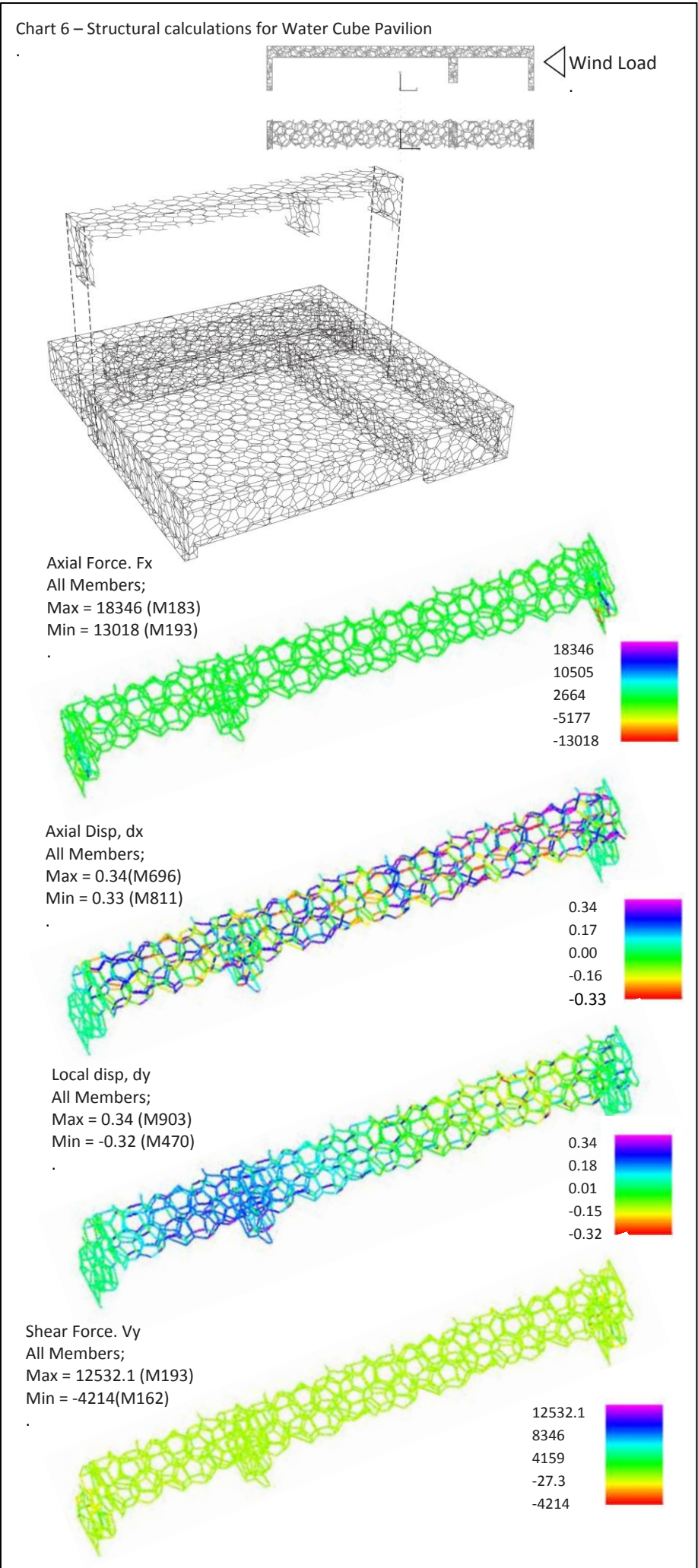


Chart 5 – Structural calculations for Water Cube Pavilion

attributes of each member, including total length, weight, grades and quantities of steel for each type of section and a price approximation. This new model contained all the dimensional properties necessary of documenting the structure of the building and it is from this model that all the construction documents were extracted. The design team calculated that it would require 112 sections to adequately document the building, thus the Bentley Structural file also became a means of recording and organizing graphically where each of the sections were taken from, as well as an order in which they would be stored in specific directories named after each individual drawing along with details and specifications.

In order to cut down on human error and facilitate the process, again this process was automated by way of a computer code. This proved beneficial when the geometry was changed or improved; the entire drawing set could be updated based on the rebuilt model in a weekend when more than 65 drawings could be generated including plans, elevations, sections and details.



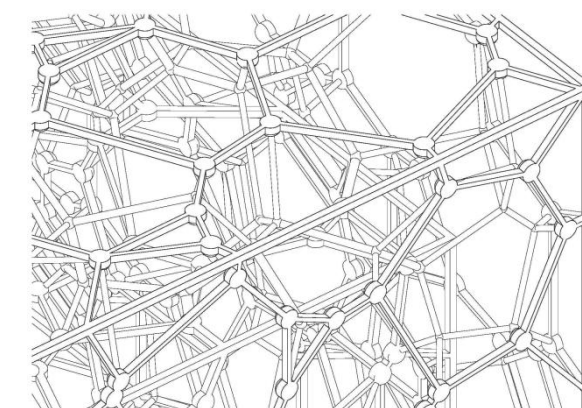
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 Total Elem 21987
 Total Prop 37
 Max Prop 500

NODES

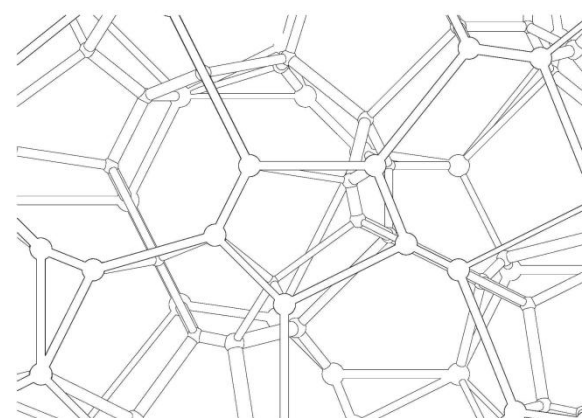
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2	88269.42	-7878.839	9214.177
3	86934.08	-14021.6	8546.503
4	85331.6	-10816.66	7745.268
5	84797.42	-6543.426	12285.59
6	84797.42	-10816.66	8012.359
7	85331.6	-6009.243	12552.68
8	87735.31	-6009.243	11350.83
9	88269.42	-7077.466	10015.55
10	88269.42	-7077.569	14822.91
11	88269.42	-11083.71	16825.98
12	87201.16	-7344.636	16291.78
13	87735.31	-10816.66	17360.09
14	84797.42	-11350.78	17093
15	84797.42	-8546.524	16291.79

Microsoft Excel - Book1.

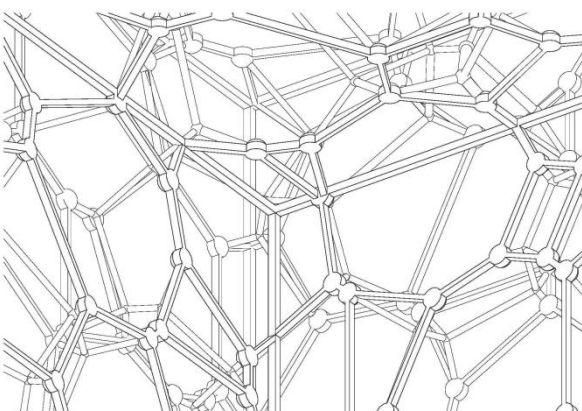
	A	B	C	D	E
1	Node Number	Node Type	Radius	Sphere (or hemisphere) Thickness	Cylinder and Top/Bottom Plate Thickness
2	1	External (Sphere+Cylinder)	411.75	17.55	13
3	2	External (Sphere+Cylinder)	411.75	17.55	13
4	3	Internal (Sphere)	184.275	9.45	-
5	4	Internal (Sphere)	218.025	12.15	-
6	5	External (Sphere+Cylinder)	411.75	17.55	13
7	6	External (Sphere+Cylinder)	411.75	17.55	13
8	7	Internal (Sphere)	271.35	14.85	-
9	8	Internal (Sphere)	271.35	14.85	-
10	9	External (Sphere+Cylinder)	411.75	17.55	13
11	10	External (Sphere+Cylinder)	411.75	13.5	10
12	11	External (Sphere+Cylinder)	411.75	13.5	10
13	12	Internal (Sphere)	271.35	12.15	-
14	13	Internal (Sphere)	147.825	6.75	-
15	14	External (Sphere+Cylinder)	411.75	24.3	18
16	15	External (Sphere+Cylinder)	411.75	17.55	13



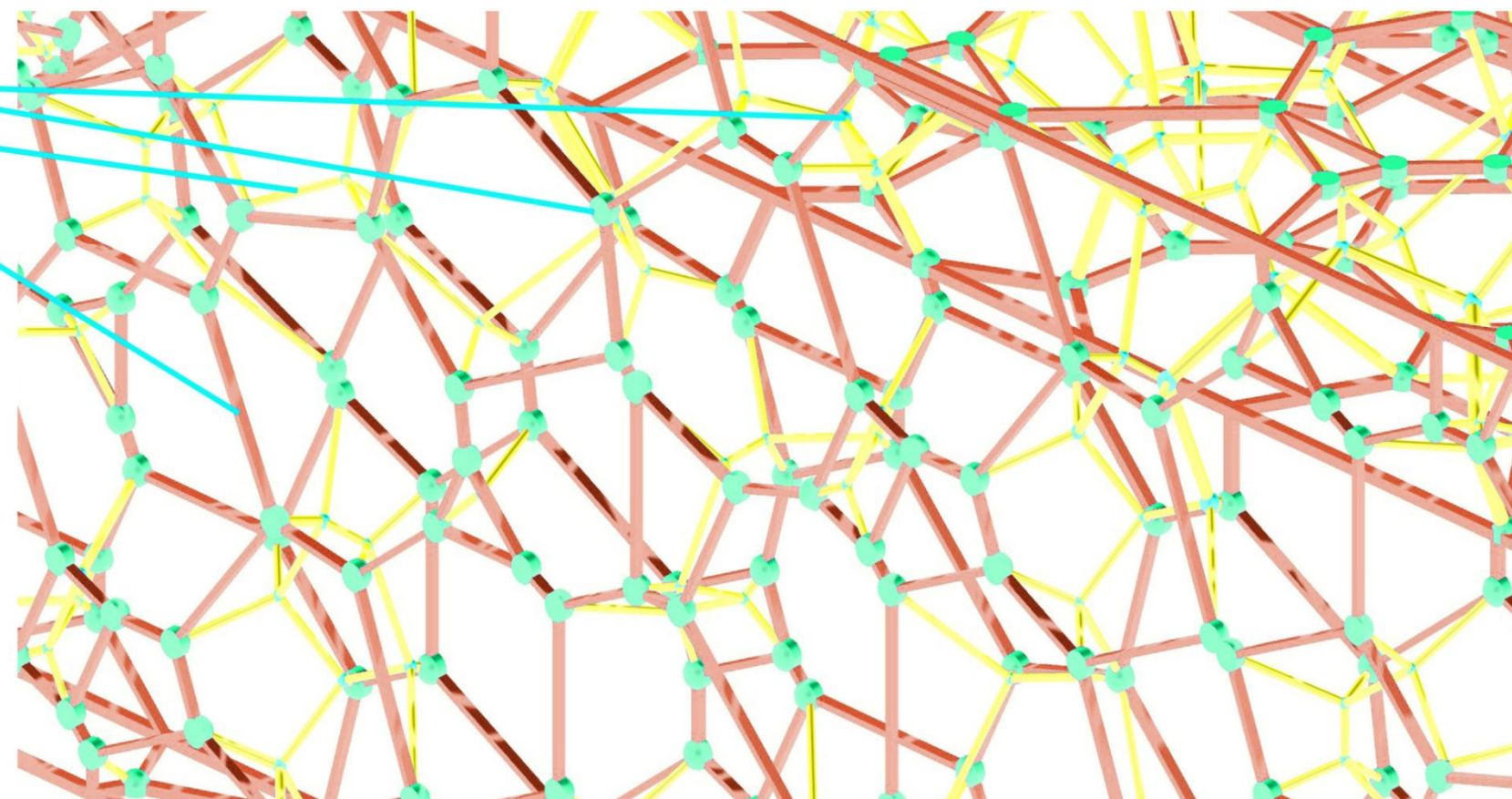
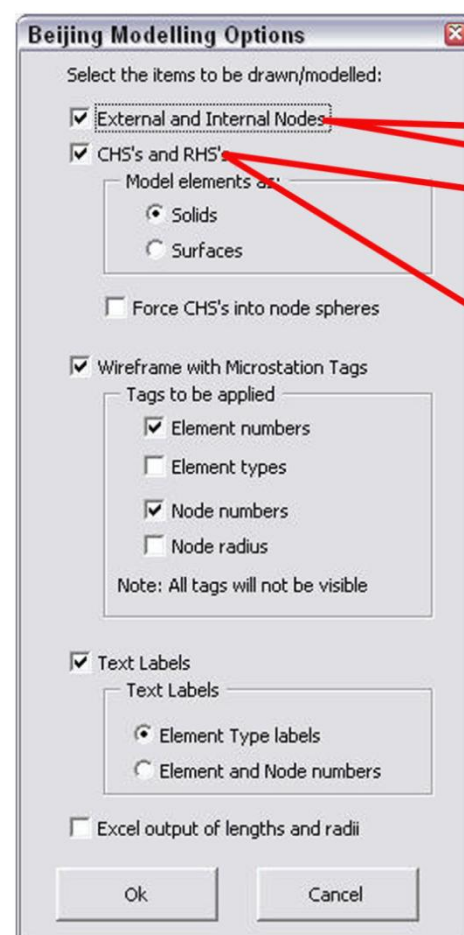
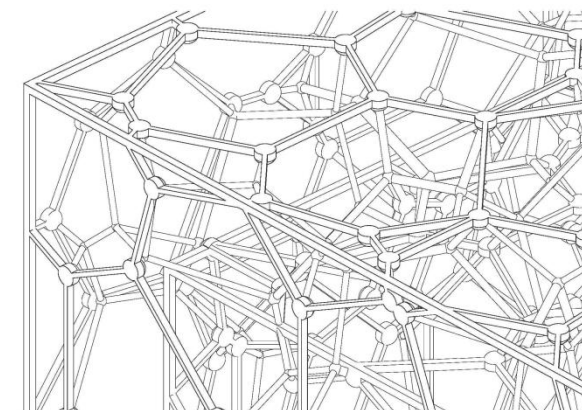
Typical Corner View 1.



Plan on typical roof frame nodes.



Typical internal corner framing.



Input Data from Microstation scripts

Chart 7 – Image representing the linking of 3d virtual model with excel sheet to extract structural specifications from the model.

Material Behaviour.

Material used as skin –

ETFE, a transparent form of Teflon, would be most efficient for such a greenhouse, removing the need for a secondary structure and providing better insulation than single glazing.

ETFE is one tenth of the weight of glass.

By cladding the ETFE cushions, Ninety per cent of the solar energy falling on the building is trapped within the structure. As swimming pools are predominantly

heating driven, this allows the scheme to harness the power of the sun to passively heat the interior area of the building and pool water. It was estimated by the firm that this sustainable concept helps to reduce the energy consumption of the leisure pool hall by 30 per cent.

The design harnesses solar energy to allow high levels of natural daylight into the building. The ETFE cladding ensures the centre will be well lit during the day, with appropriate levels of internal daylight, visual connection and visual connection and visual comfort.

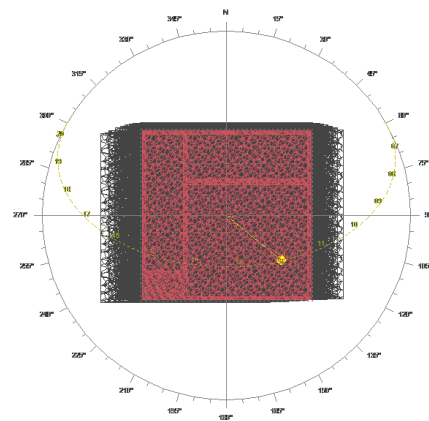
Building Envelope

The dual-ETFE cushion envelope allows the WaterCube to achieve great thermal efficiency. The envelope is so

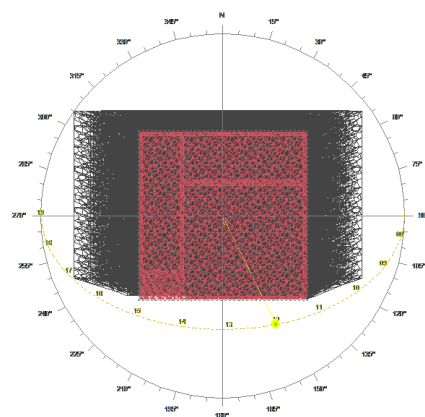
well insulated that the design may create an annual net heat gain. The concrete structures containing the pools will act as thermal masses, retaining heat during the warm days and releasing it over the course of cool nights.

The envelope has three modes of operation, allowing the building to adjust to winter, summer and mid-season conditions. These changes are achieved by varying the amount of light and solar radiation entering the interior, by varying the translucent and transparent foils. Hence, the level of natural day lighting is also easy to control and alter rapidly. There is no need for artificial illumination during daylight hours. At night, the building is lit from within giving it a soft glow and dominant place in the landscape.

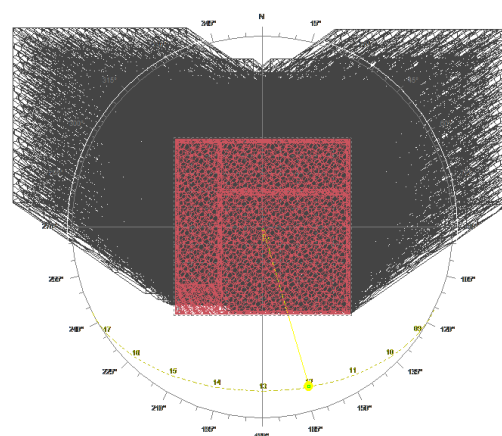
Opening and closing the internal foils by varying their internal pressure is what allows for such flexibility. Dappled lighting effects can be achieved, or light can be directed only to certain areas. Additionally, the entire envelope can be 'turned off' to achieve the lighting situation that works best for televised events.



Summer Solstice - 9am-7pm.



Equinox Condition - 9am-7pm



Winter Solstice - 9am-7pm

Figure 58 - Analysis based on sun movement

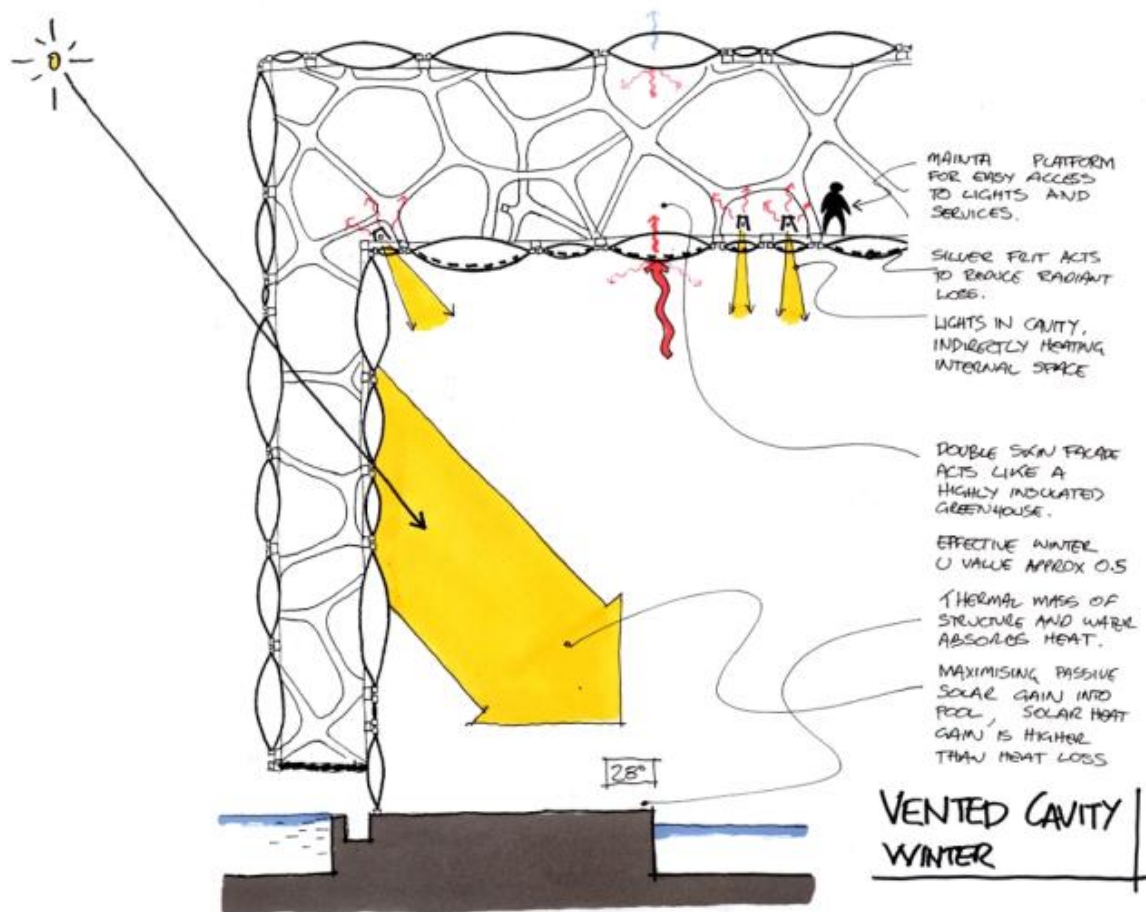


Figure 61 –Original sketch by the firm showing how the skin works in winter.

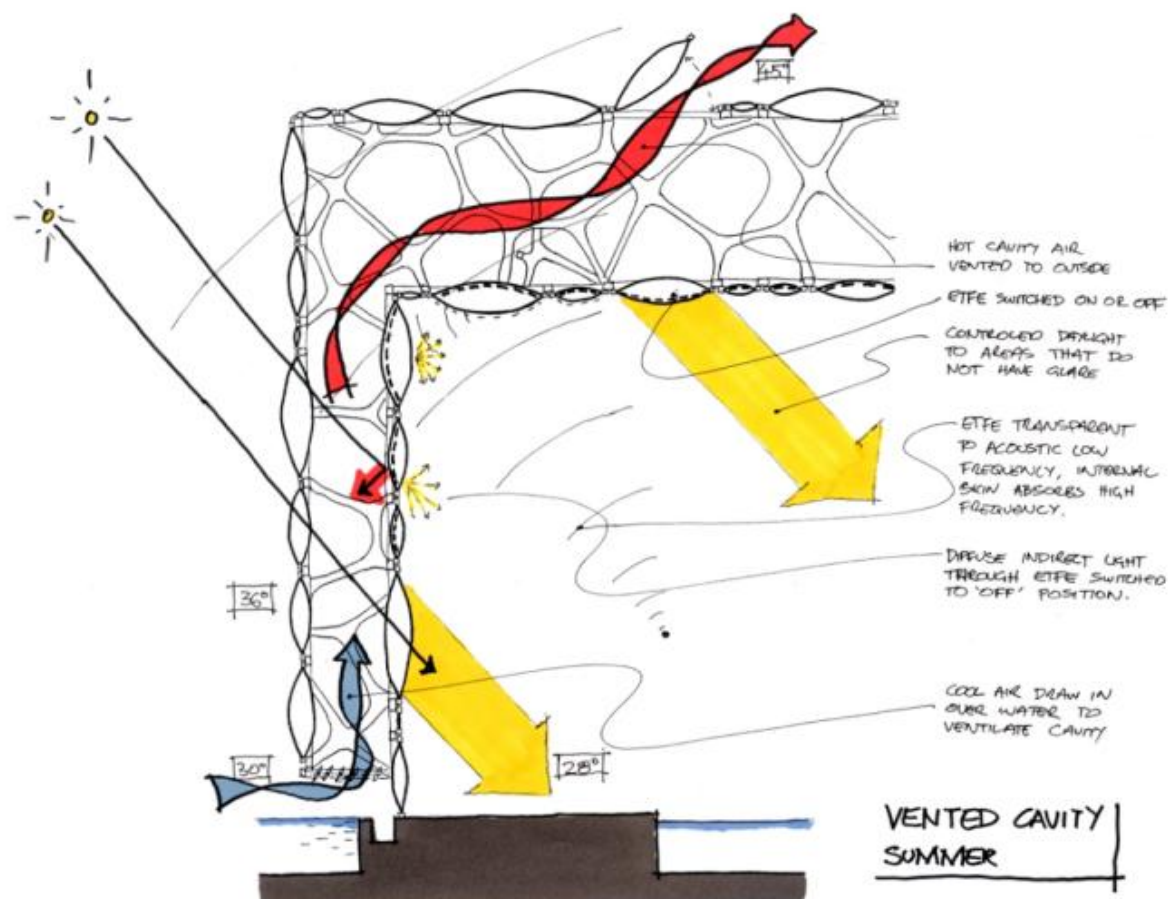


Figure 62 –Original sketch by the firm showing how the skin works in summer.

Installation Methodology

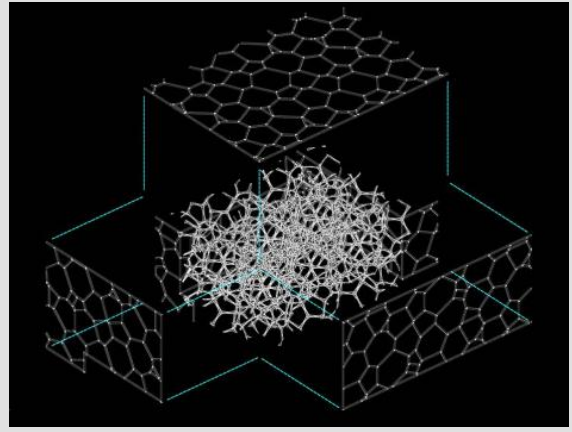


Figure 63 –Computer image showing the extraction of drawings from a virtual model

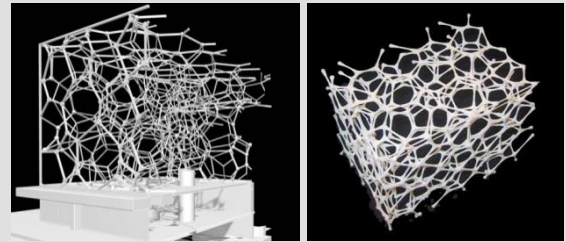


Figure 64 –Physical models of weaire phelan structure

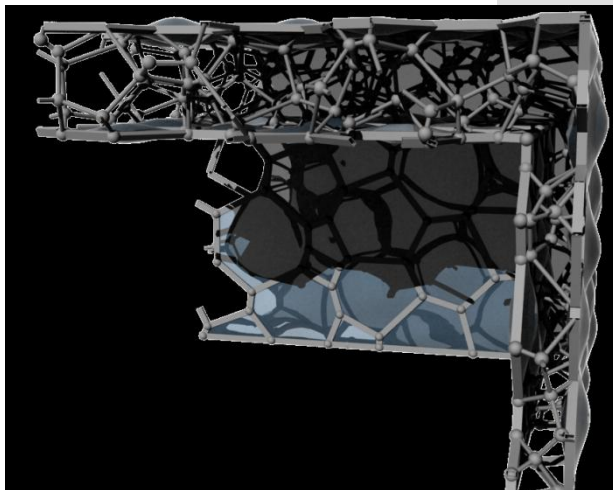


Figure 65 –Detail wall section rendering.



Figure 67 –Image showing the installation of ETFE sheets over the structure..

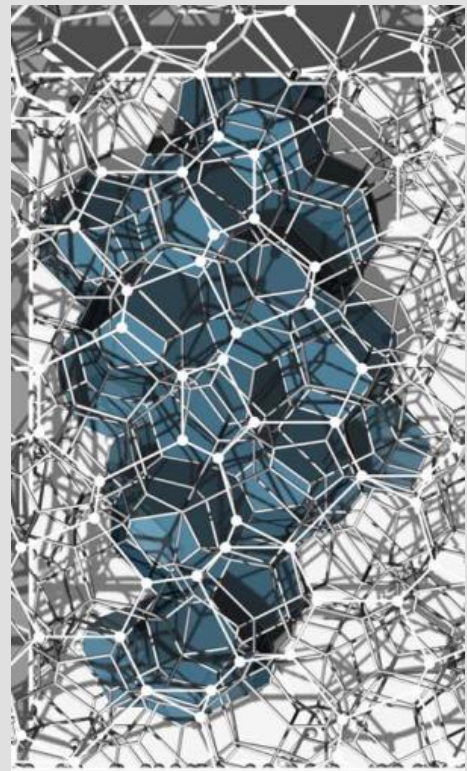


Figure 66 –Physical models of weaire phelan structure

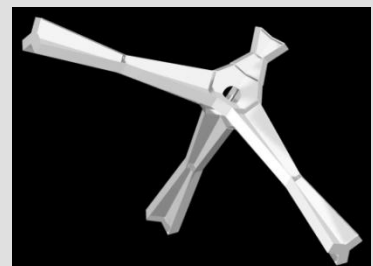


Figure 68 –A virtual model of the connecting details.



Figure 69 – Interior View.

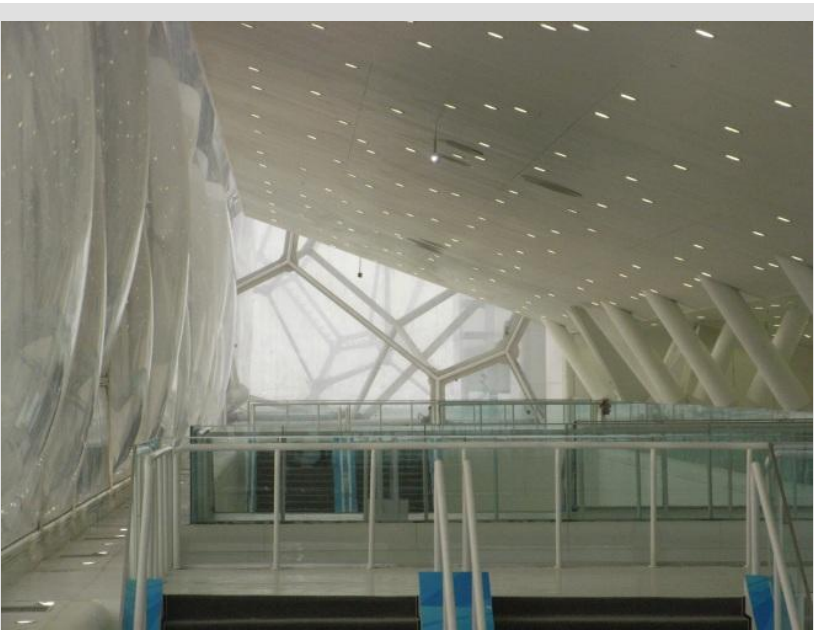


Figure 70 – Interior View.



Figure 71 – Interior View.



Figure 72 – Interior View.

Interior Views of Water Cube Pavilion.



Figure 73 – Interior View.

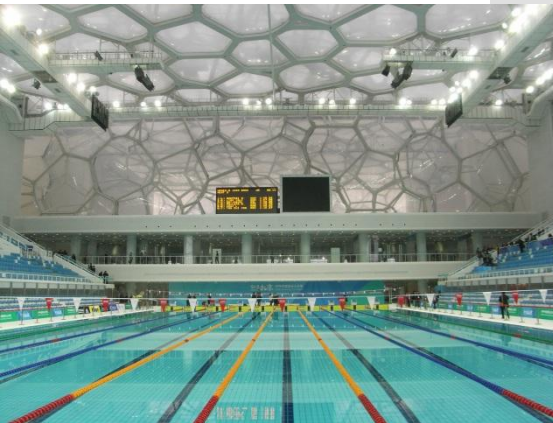


Figure 74 – Interior View.



Figure 75 – Interior View.



Figure 76 – Interior View.



Figure 77 – Interior View.



Figure 78 – Interior View.



Figure 79 – Interior View.

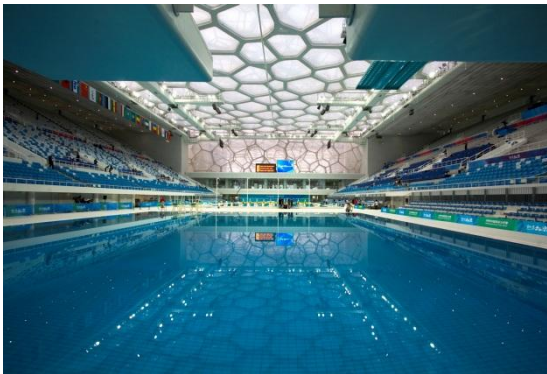


Figure 81 – Interior View.



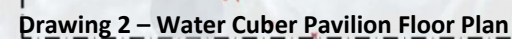
Figure 80 – Interior View.

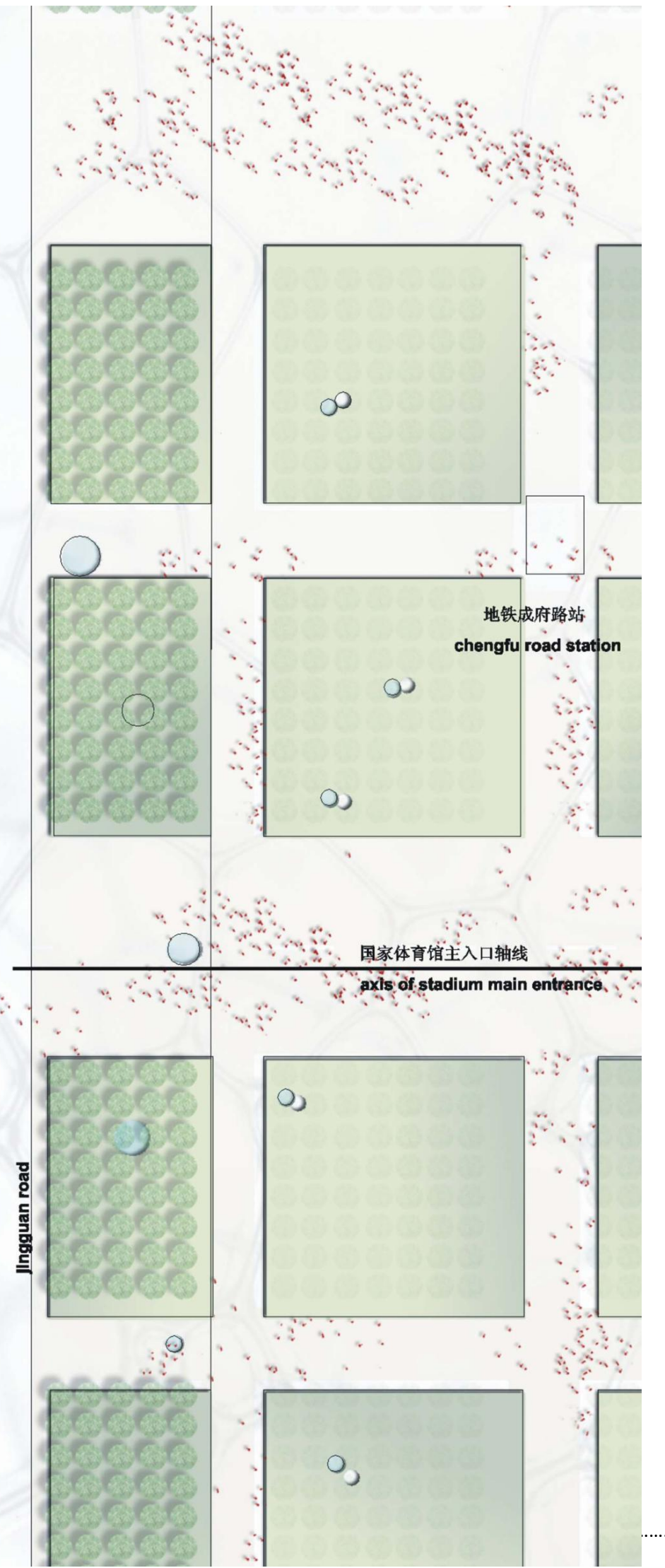
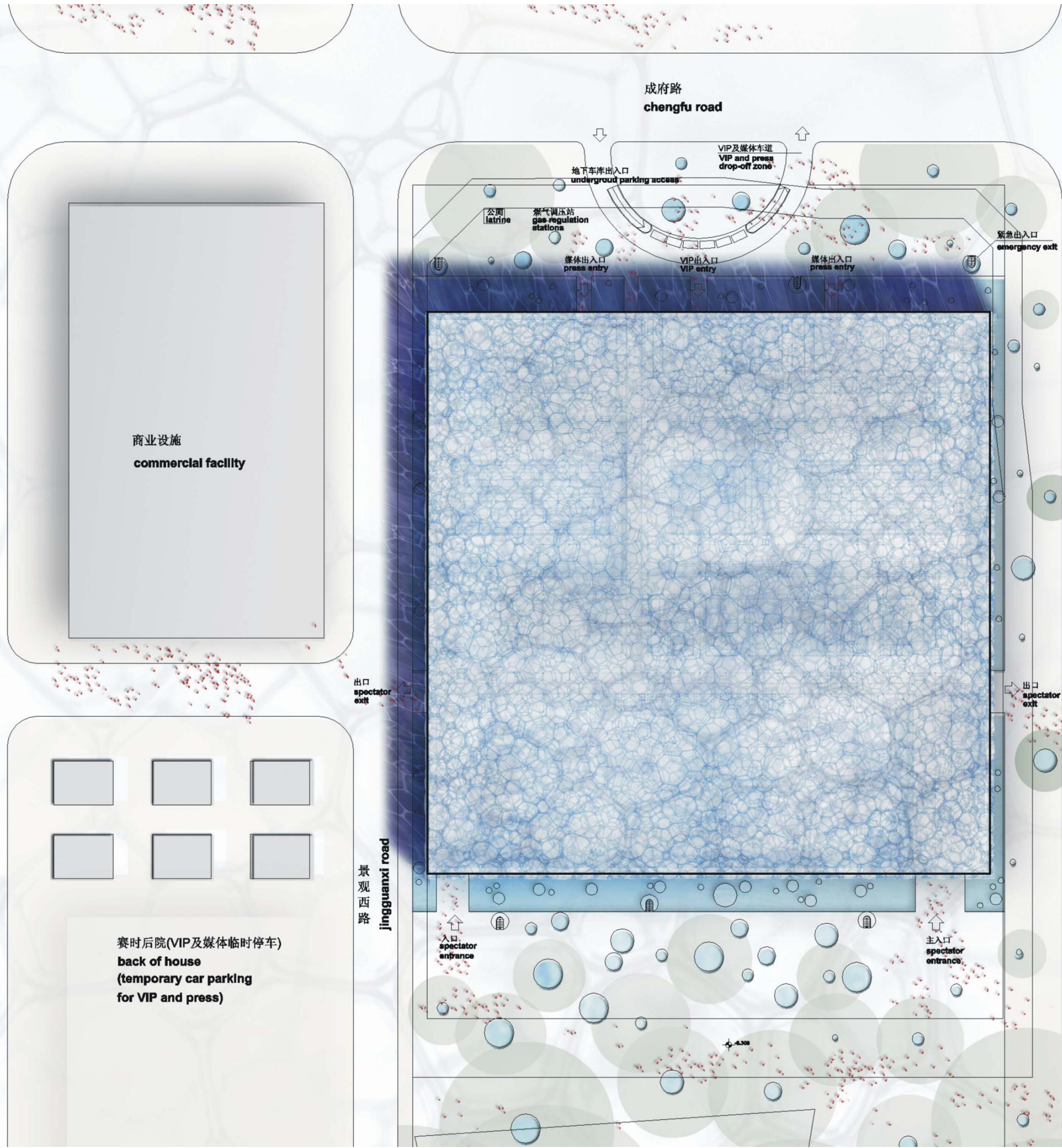


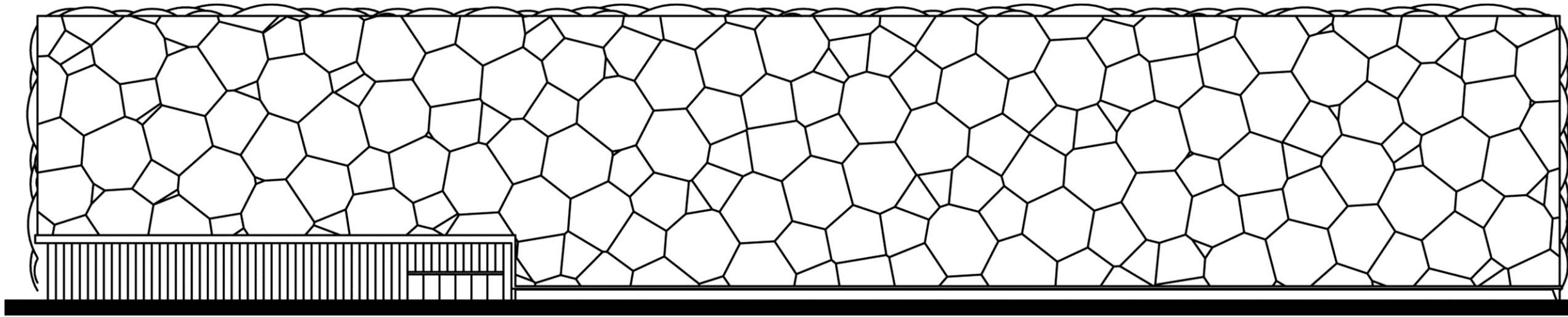
Figure 82 – Interior View.



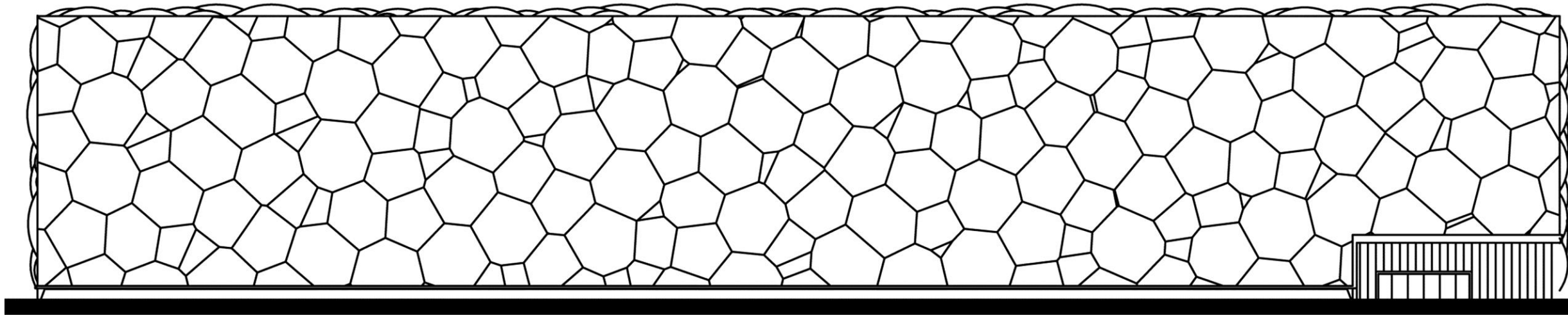
Figure 83 – Interior View.



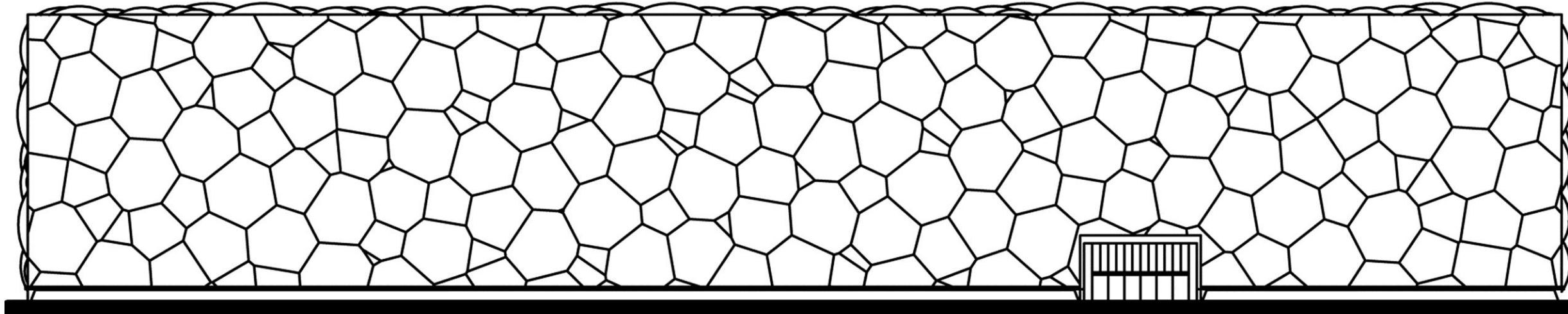




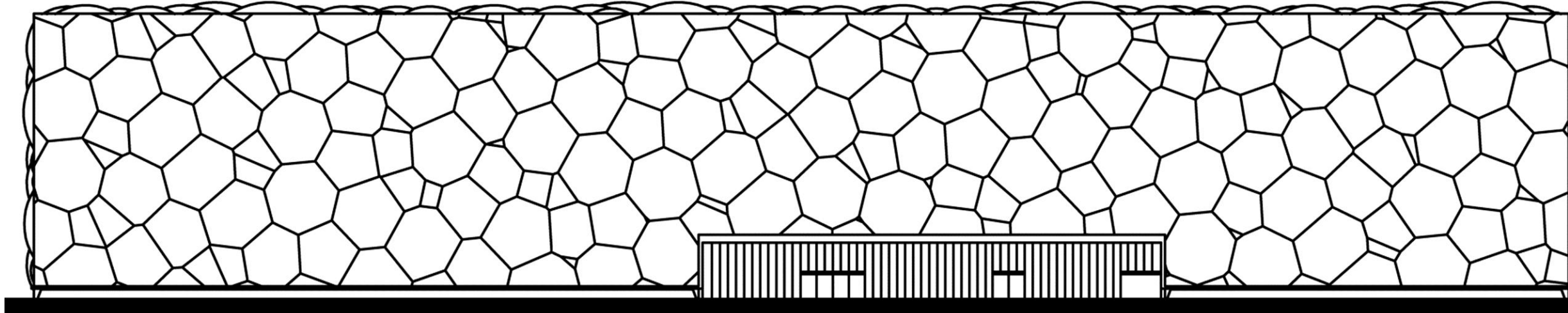
Drawing 5 – East Elevation



Drawing 6 – South Elevation



Drawing 7 – West Elevation



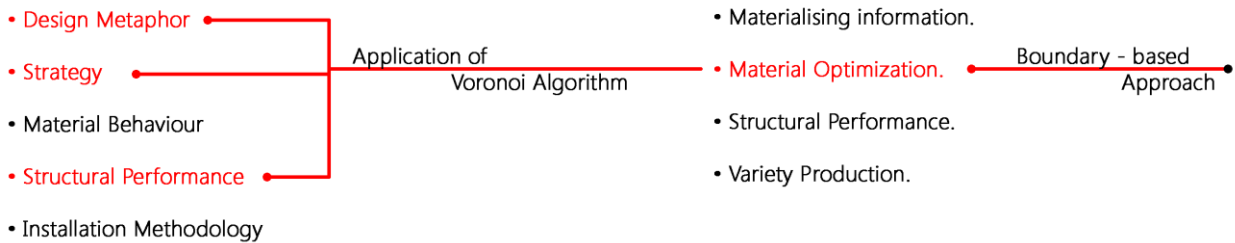
Drawing 8 - North Elevation



Based on the study of Water cube pavilion, in this project Voronoi algorithm is applied as a design metaphor, but except that it also helps in material optimization.

- Type of Voronoi Algorithm used – Centroidal Voronoi Algorithm.
- Application – 3 dimensional.
- Maximum affected category
 - Strategy.
 - Material Optimization.

Defined Approach based on analysis – Boundary based approach.



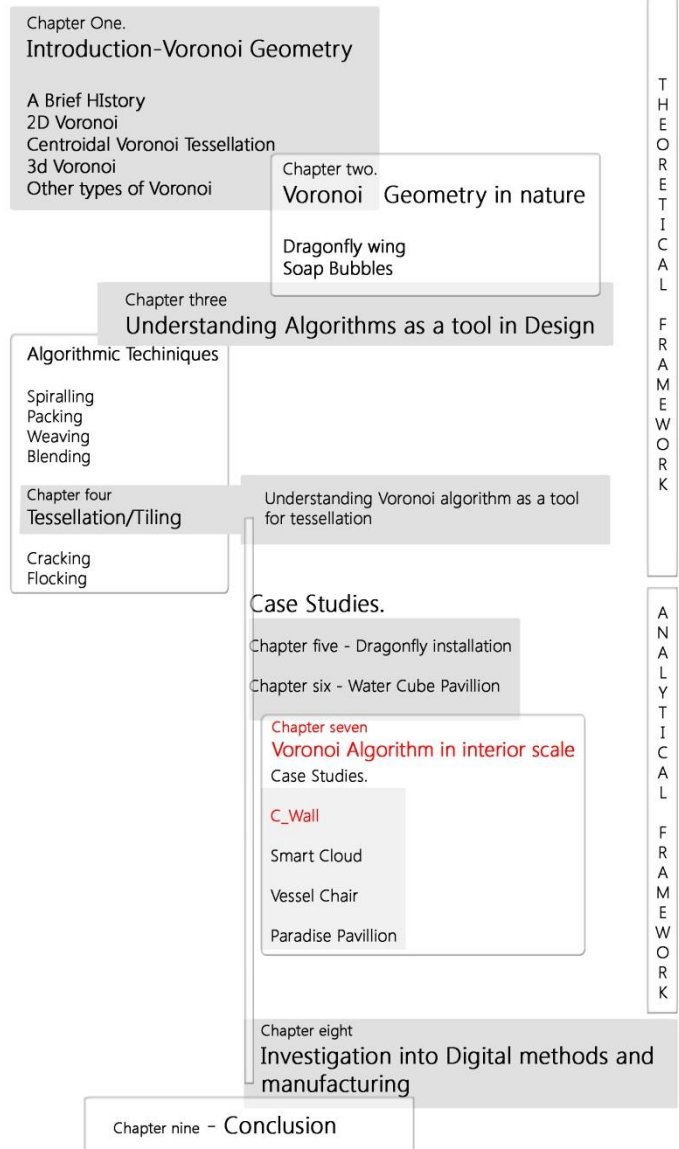
In Water Cube Pavilion the application of Weaire-Phelan structure was initially for the purpose of achieving a particular design metaphor, but it not only helped the project to achieve a desired look but it also affected the overall strategy of the project.

Eventually it resulted into structural performance of strength for the skin.

It optimized the material that was used for the outer structure, though not desired initially but this variable can be seen as the most affected due to the application of Voronoi algortihm.

Thus in this project the boundary based approach is very evident.

This project is also a fine example for the application of Centroidal Voronoi Algorithm.



C-WALL

Year – 2006

Location – Banvard
Gallery. Knowlton School
of Architecture. Ohio State
University, Columbus, Ohio

Size – 12' x 4' x 8'.

C-Wall in
interior scale can be seen
as a partition and also as a
lighting element.

In this project
Voronoi Algorithm affects
mostly in three categories,
which are

- Structural
Performance
- Material Behaviour
- Materialising
Information

Though the
application of algorithm
affects these areas in
design but the category in
which the algorithm is
used strategically is of
materialising information.

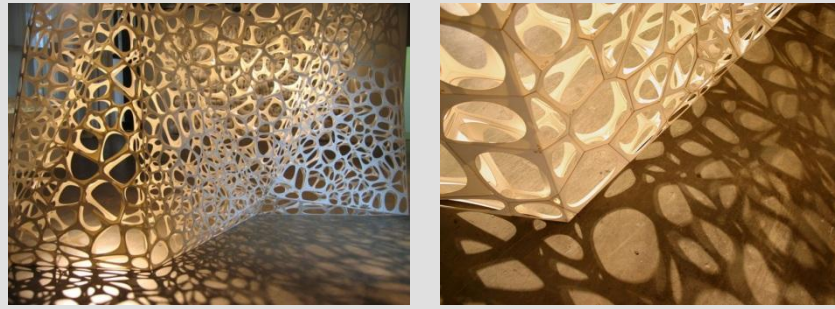


Figure 84 and 85– C_Wall creating patterns by shadows



Figure 86 – An image of C_Wall

Structural Performance

Built of thin paper and weighing very light, the structure exhibits an extremely high strength-to-weight ratio. In addition the wall produces interesting patterns of light and shadow that are based on the differentiated pattern of cell sizes.⁵³

The variable developed to measure structural performance in this project is of weight and strength.

Material optimization

In this project material Optimization is a direct by-product of such a process. Material directly affected the weight variable of the design, and thus this variable was in indirect relation with the structural performance and was also a compulsion in this case.

Application of Voronoi Algorithm

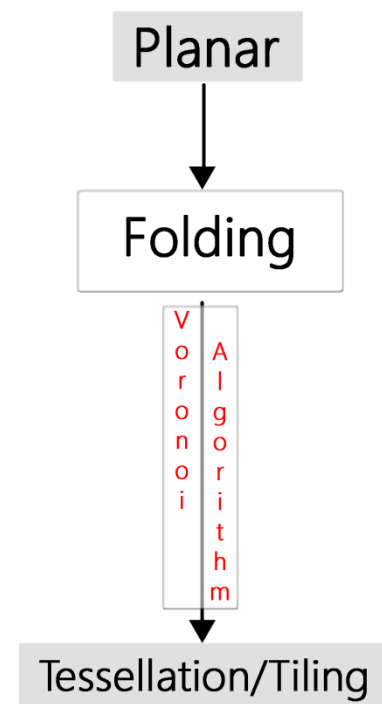
C_Wall is the latest development in the architect's on going area of research into cellular aggregate structures. The project employs the Voronoi algorithm.⁵⁴

In this case, the Voronoi algorithm facilitates the translation and materialization of information from particle simulations and other point-based data. Through this operation, points are transformed into volumetric cells, which can be unfolded, CNC-cut, and reassembled into larger aggregate.

This project dealt with two aspects of algorithms which are

- Tessellations/tiling
- Folding

Folding turns a flat surface in three- dimensional one. When folds are introduced into otherwise planar materials, those gain stiffness and rigidity, can span distance, and can often be self-supporting. Folding is materially economical, visually appealing, and effective at multiple scales.



⁵³ Digital Fabrication – Lisa Iwamoto

⁵⁴ http://matsysdesign.com/2009/06/19/c_wall/



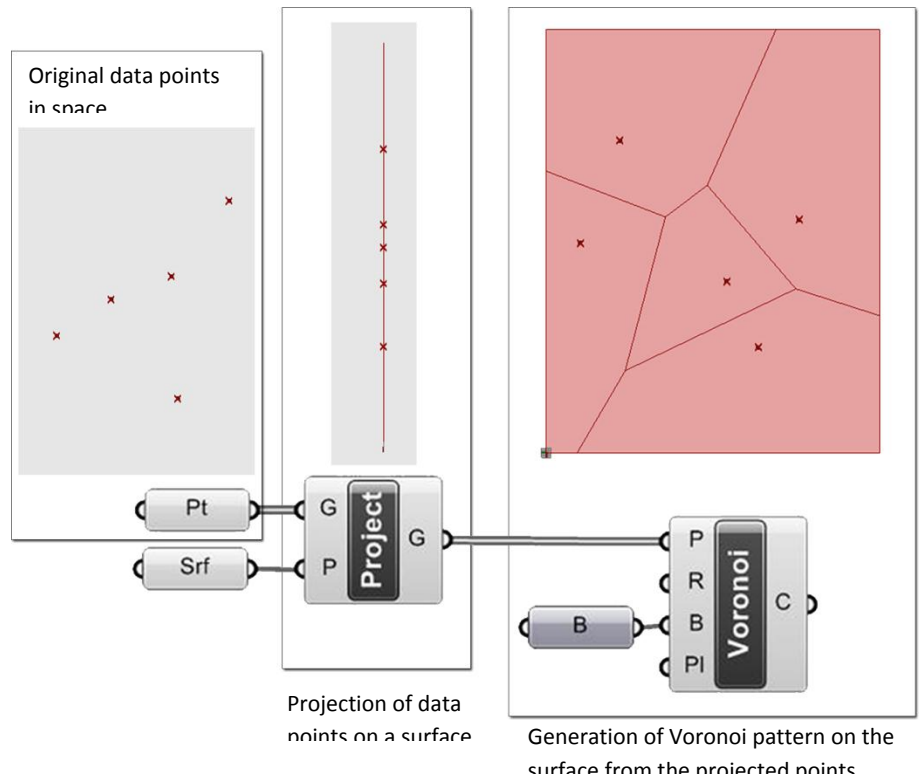
Figure 87 – Image showing the site and C_Wall as an interior element

In between folding and tiling, there is also a very interesting implication of the process of projection.

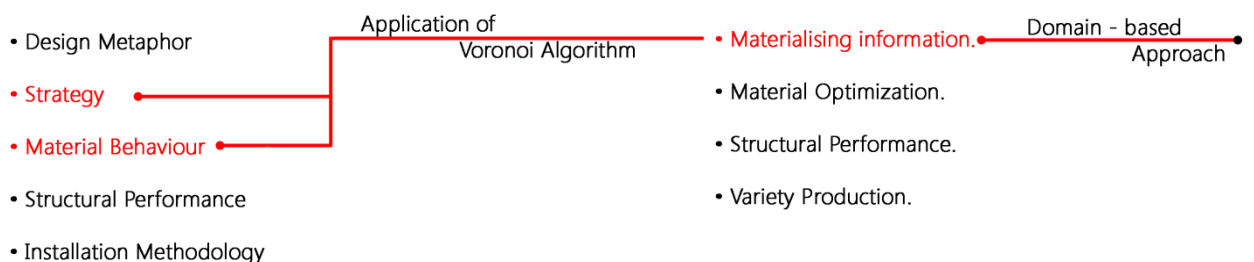
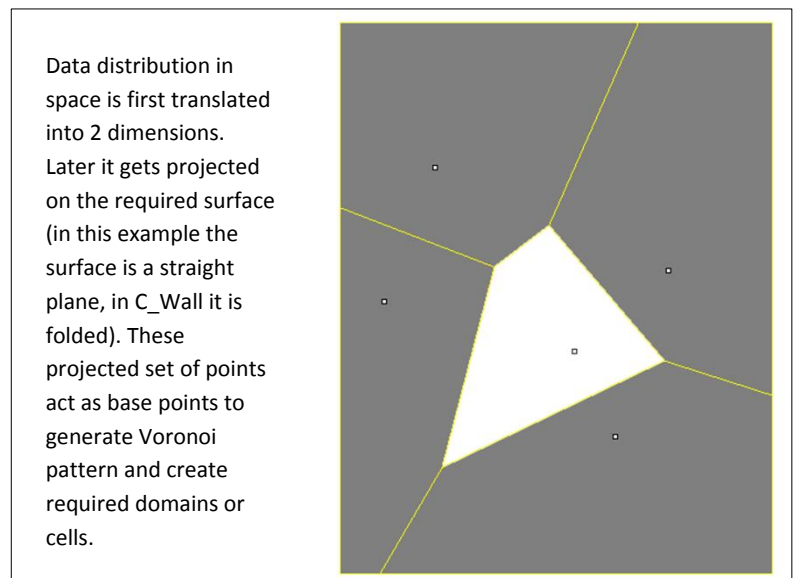
Before Voronoi algorithm acts as a translator of the point based information into cells or domains, the data points needs to be projected on the folded mass model. This projection acts as a mediator or controller between folding and tiling

Translation from particle-simulation and point-based data results in voids. Such voids in this analysis are understood as domains.

Particle simulation or the achieved point-based data can be achieved from any requirement. The overall process thus revolves around information which is an input of domain and thus the resultant boundary act as a by-product in the output.

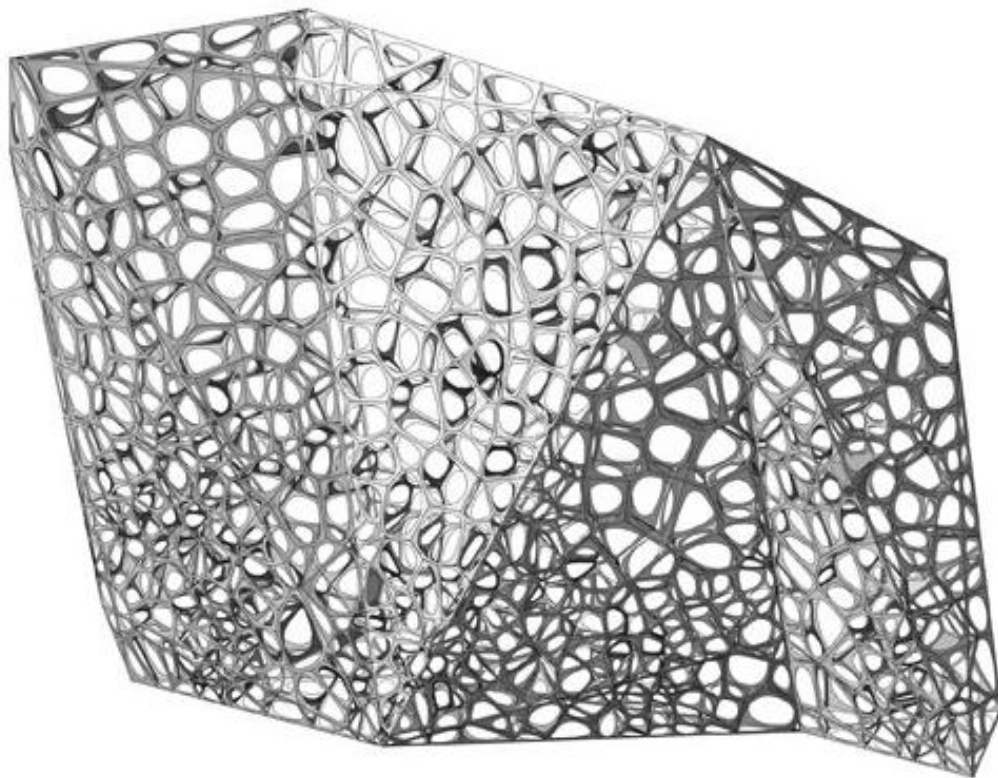
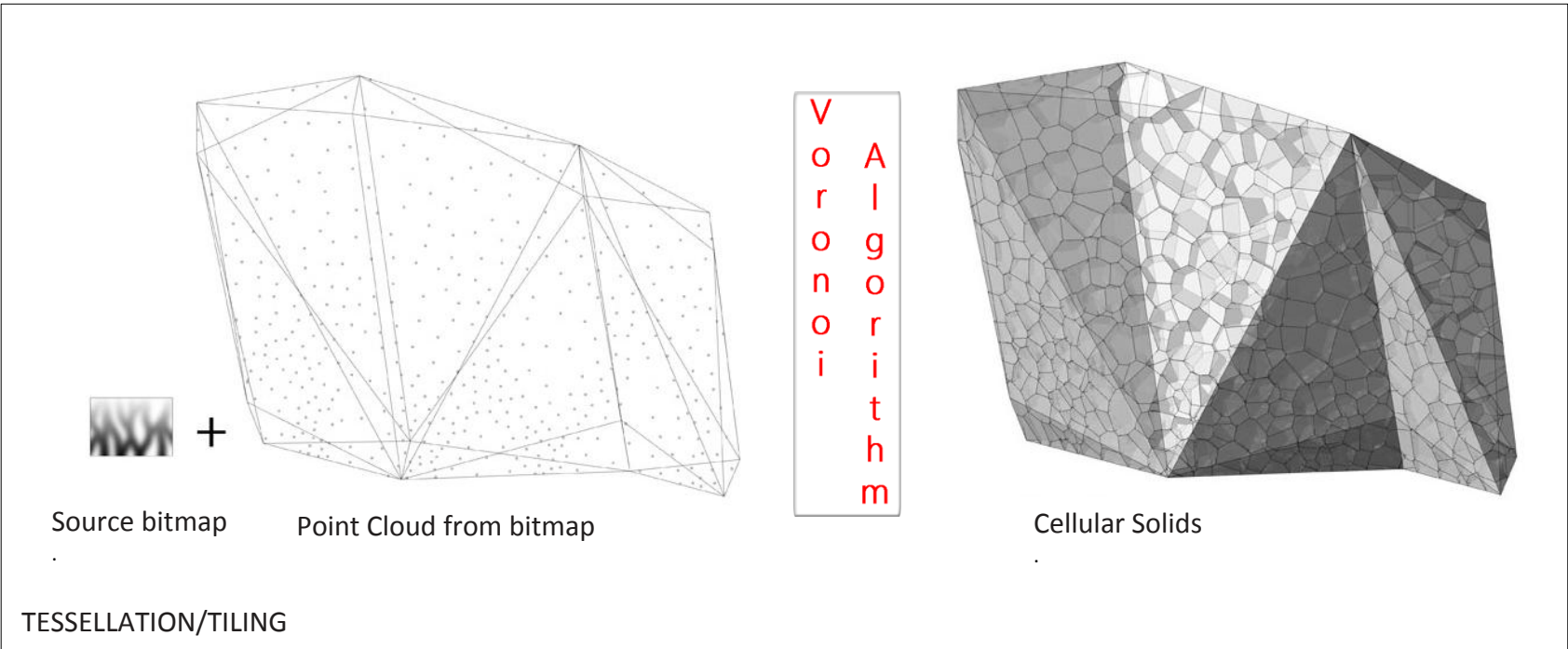
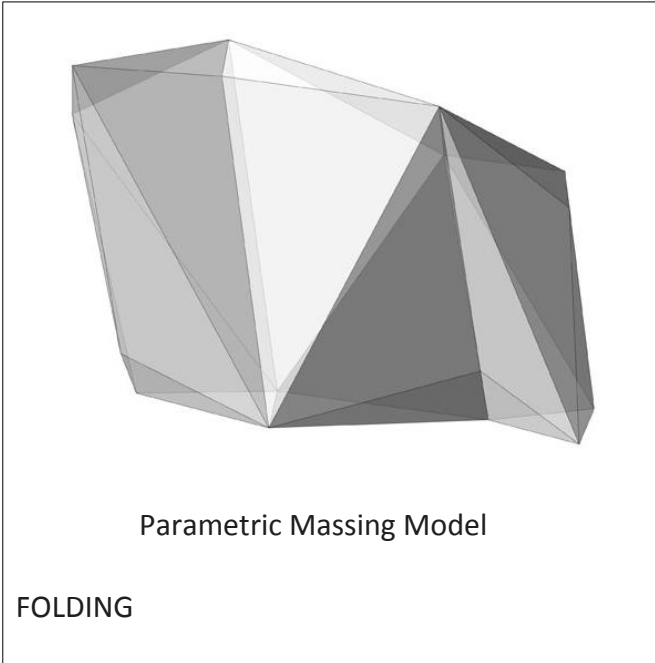


Thus this is domain-based approach.

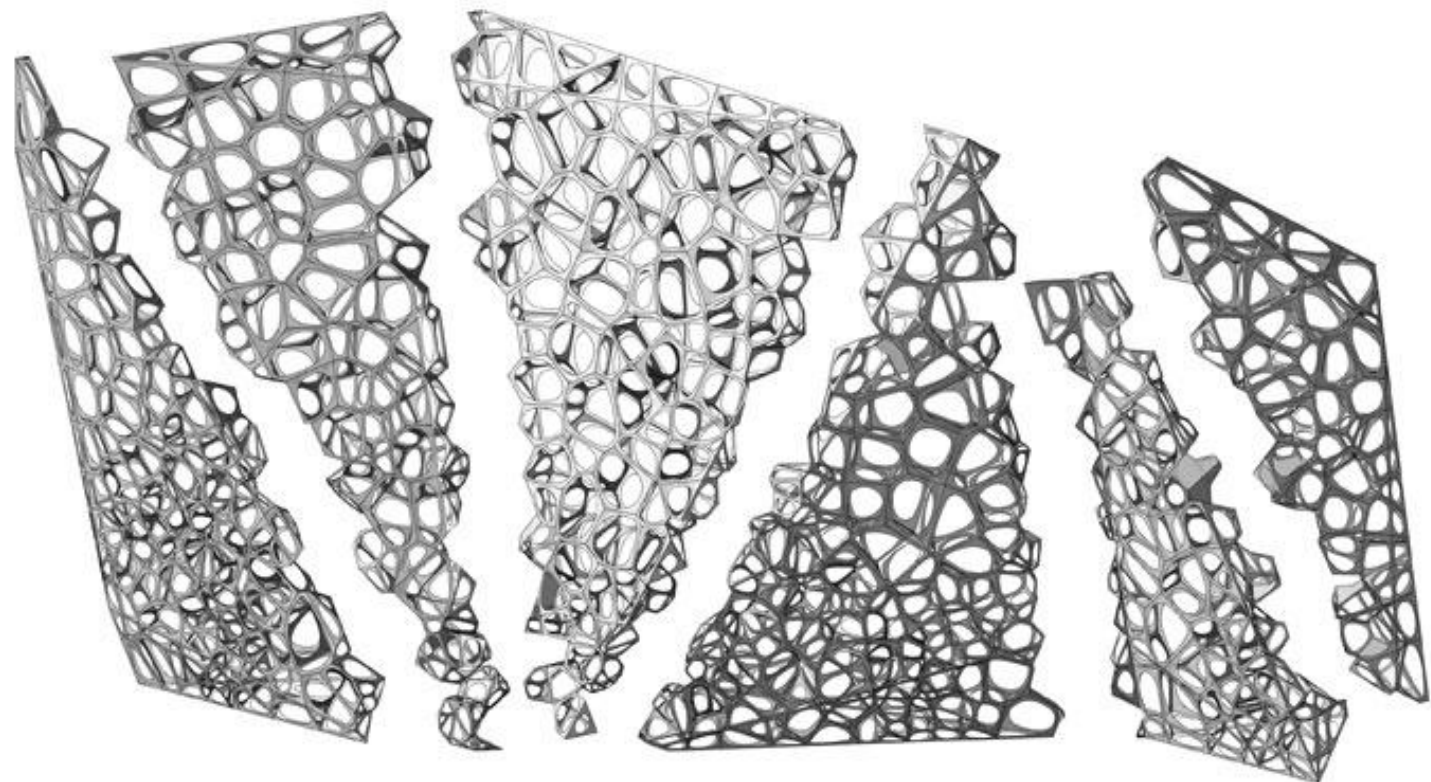


Materialising information

Translation of information from particle simulation.



Voiding



Panelisation

Chapter One.
Introduction-Voronoi Geometry

A Brief History
2D Voronoi
Centroidal Voronoi Tessellation
3d Voronoi
Other types of Voronoi

Chapter two.
Voronoi Geometry in nature

Dragonfly wing
Soap Bubbles

Chapter three
Understanding Algorithms as a tool in Design

Algorithmic Techniques

Spiralling
Packing
Weaving
Blending

Chapter four
Tessellation/Tiling

Cracking
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Understanding Voronoi algorithm as a tool
for tessellation

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Chapter five - Dragonfly installation

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Chapter seven
Voronoi Algorithm in interior scale

Case Studies.

C_Wall

Smart Cloud

Vessel Chair

Paradise Pavillion

Chapter eight
Investigation into Digital methods and
manufacturing

Chapter nine - Conclusion

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The smart Cloud.

This project presents the process by which Cook+Fox Architects responded to a design challenge that was part metaphorical and part practical.

The project involved providing an environmental response to the natural world existing almost 800 feet above the ground, on the second-highest occupiable floor of New York City's second-tallest building.

Design Metaphor

The Smart Cloud was conceived as an acknowledgment of this history and the natural occurrence of this form in nature.



Figure 88 – Image showing the site

*The sunlight's glory on the violet shoals,
the cities' glory as the sunlight wanes,
kindled that restless longing in our souls,

to plunge into the sky's reflected flames.
The richest cities, the greatest scenes, we found
never contained the magnetic lures,
of those that chance fashi oned in the clouds.*
Charles Baudelaire (1861)

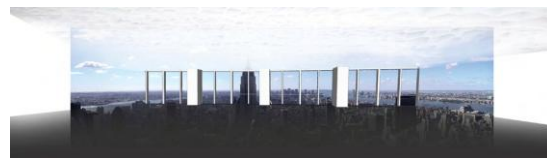


Figure 89 – An abstract representation of the metaphor

Although the cloud concept began as a metaphor, it quickly became something more tangible, mappable, and solvable

Environmentally-responsive features at the Bank of America Tower at One Bryant Park are expected to make it the first LEED-Platinum high-rise in the world. The fiftieth floor was conceived as a headquarters for the fashion designer Elie Tahari, the south facing portion of the floor plate was to house a highly adaptable showroom that needed to be adaptable to complement and enhance each season's particular aesthetics. Additionally, the ceiling in the showroom space needed to allow for optimized height in an environment where structural, mechanical, electrical and sprinkler systems were all designed to be concealed. A combination of numerous computer-aided design scripts took into account various input variables and finally led to the generation of a Smart Cloud.⁵⁵



Figure 89 – Reference image for the project used by the designers showing Voronoi pattern

⁵⁵ Report - ACADIA 08 › Silicon + Skin › Biological Processes and Computation

Application of Voronoi geometry for false ceiling

This project represents an intersection of mathematics, nature, poetry, and the pragmatics of architectural design. It emerged from a commission to design a showroom and office space for a major label fashion designer that also needed to function as an environmental response to the natural world existing at nearly 800 feet above the ground. At that height, the immediate surroundings are atmospheric, consisting basically of the horizon and a few speckles of Manhattan’s neighbouring building tops. The project therefore sought to evoke a relationship between exterior and interior by conjuring a ceiling pattern that recalled the look and feel of clouds, those accumulations of water vapour whose chance formations had once so entranced Baudelaire (Figure 1).

Strategy

The application of Voronoi algorithm was to serve as a base for the representation of clouds.

Inside, a showroom was to be designed along the south facing portion of the floor plate to highlight the designer’s latest fashions. Increased ceiling height was much desired for this space, but ceiling beams were spread out at odd angles and sizes to provide for an efficient load distribution, while the showroom’s increased load capacity necessitated the use of an overhead air distribution system.

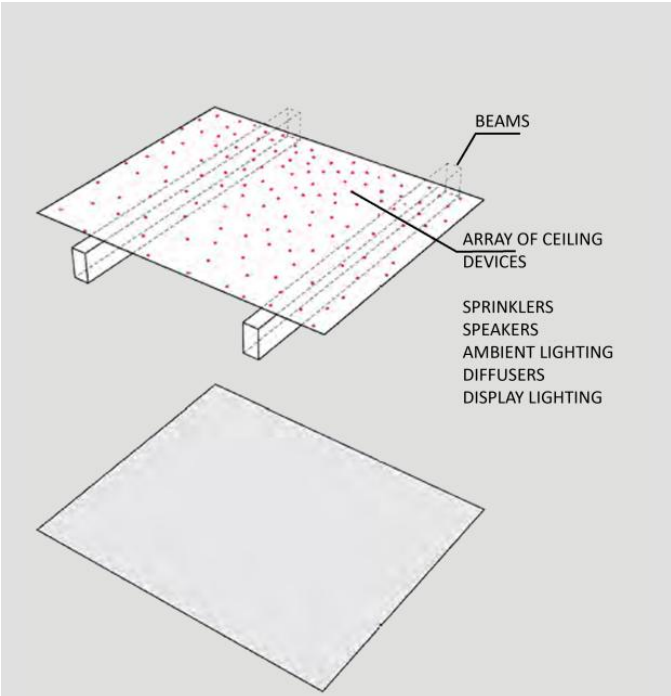


Figure 90 – Diagram representing the initial set of points for the generation of Voronoi pattern

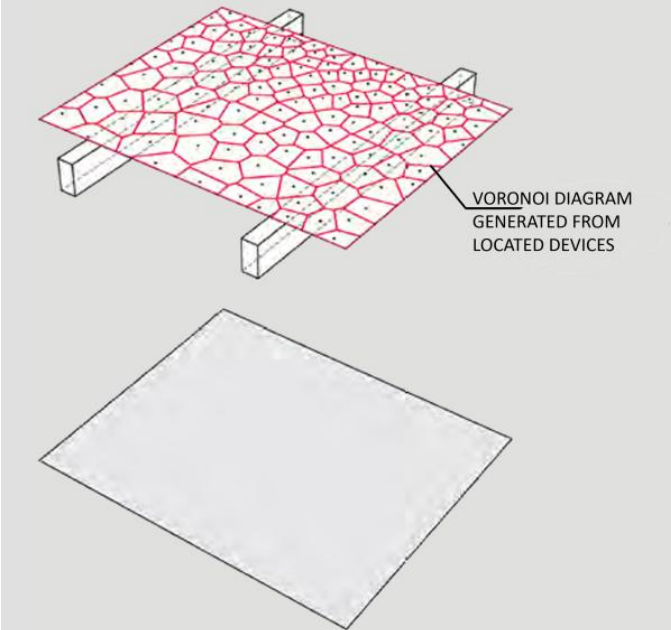


Figure 91 – Voronoi diagram generated from located points (devices).

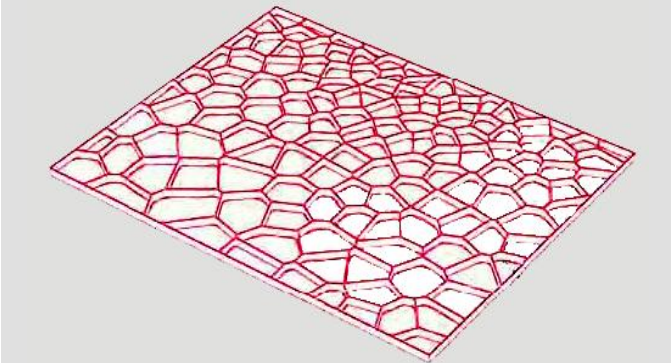
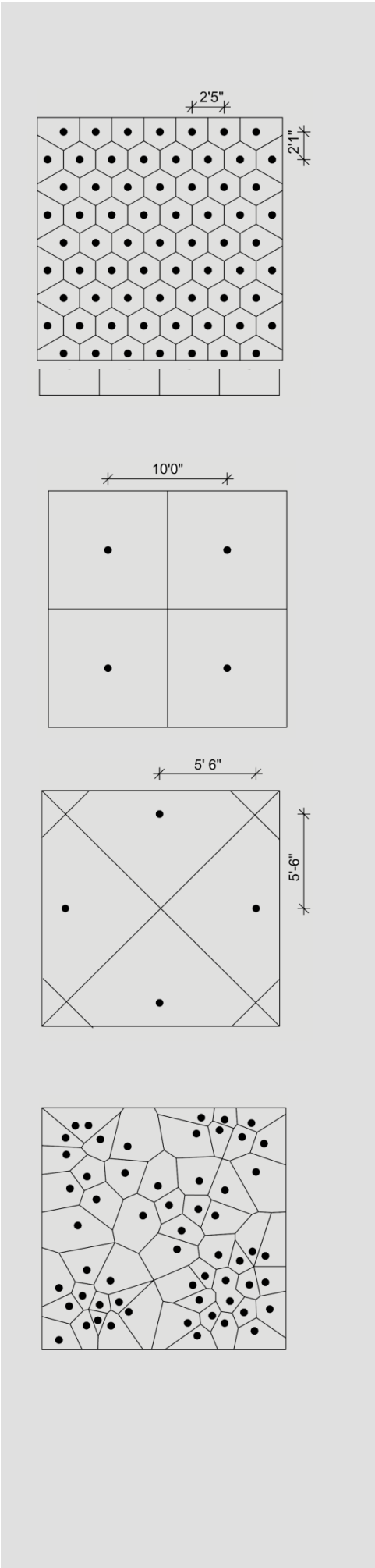
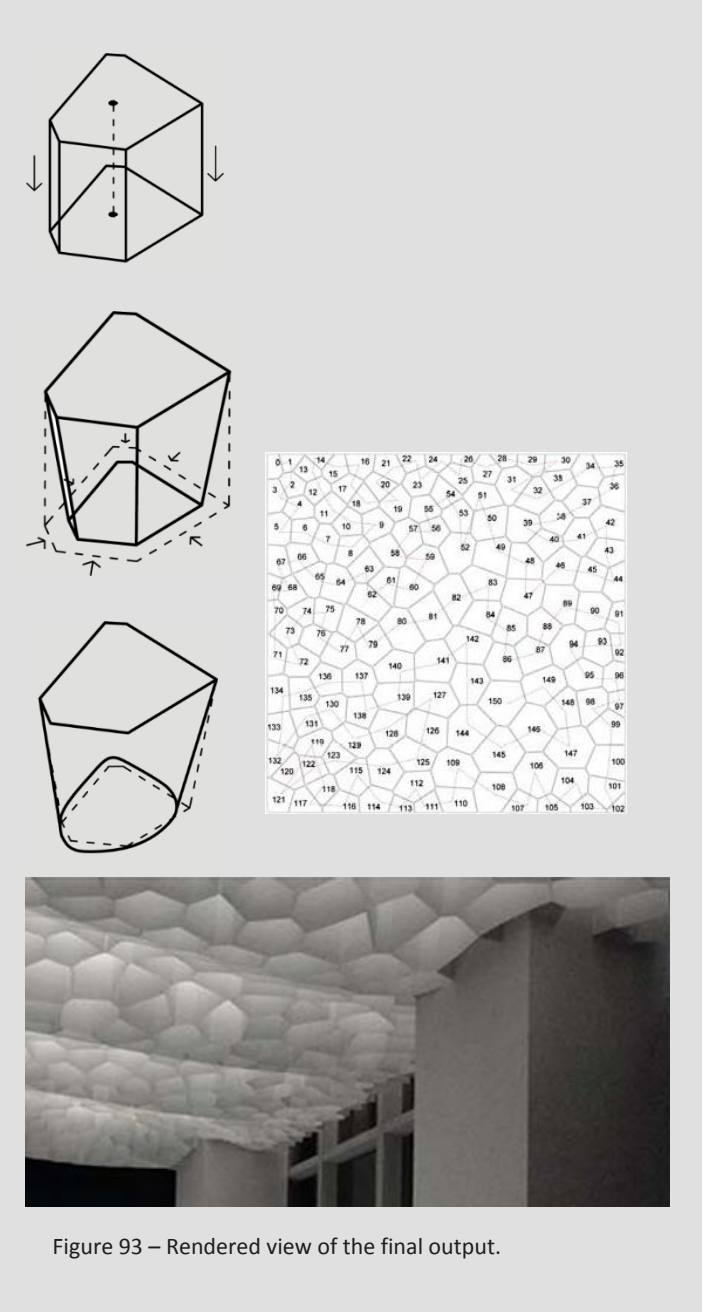


Figure 92 – The final output.

These systems were intended to be entirely concealed by a traditional hung ceiling of 9'6". Substituting this with a dynamically formed ceiling that could hug the structure above—rolling, undulating, and billowing out where it needed to accommodate structure, ductwork, and lighting—would contribute visually to the atmosphere, but would also perform responsively to meet the needs of the varied program requirements of the showroom space. As both metaphor and structure, the Smart Cloud was a constant reminder of our relationship to the natural world of which this building and its fashion showroom are a part. But it also placed this interior in a telling visual relationship to the history of modernity, within which clouds have long been a central motif



Installation Methodology

For various reasons, this project was not ultimately executed, instead remaining in the conceptual realm of the clouds. However, it makes an important contribution through the integration of scripting for parametric computer aided design and manufacture, natural concepts, and sustainability of form, material, and function. Manifested through an evocative metaphor, this cloud-like pattern found logic to its structure in the form of Voronoi tessellations. Beyond this architectural weather phenomenon offering a practical solution for a very real concern, the Smart Cloud functions as a harmonious buffer between the interior and the exterior, encouraging a constant dialogue between architectural design and a high-altitude ecosystem, offering a permanent reminder of our interconnectedness with all other things.

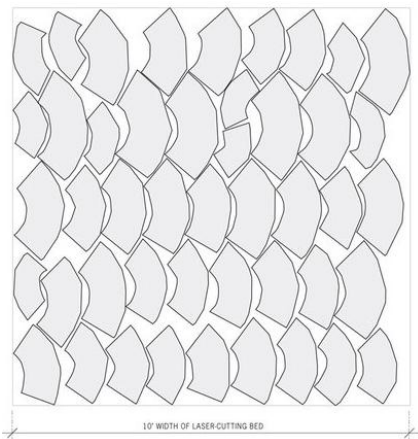


Figure 94 – Pieces of individual cell before assembly

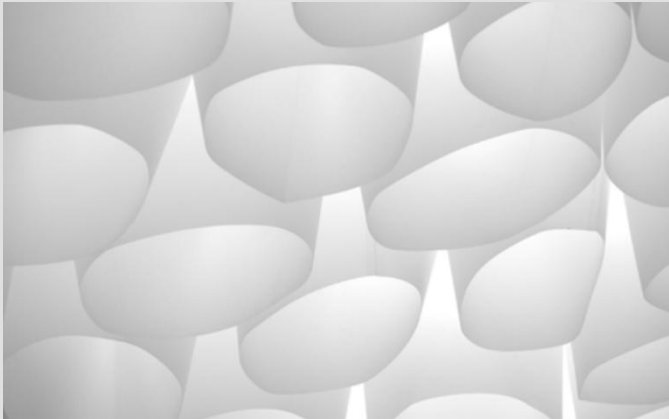


Figure 95 – Prototype of smart cloud



Figure 96 – Prototype of smart cloud. Assembly

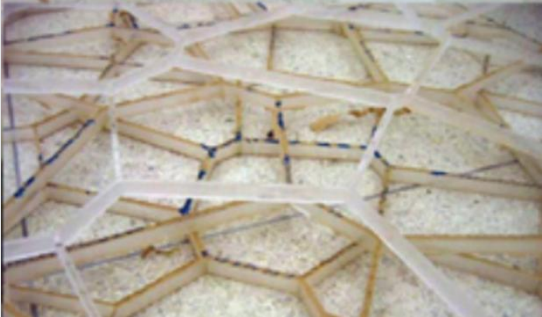
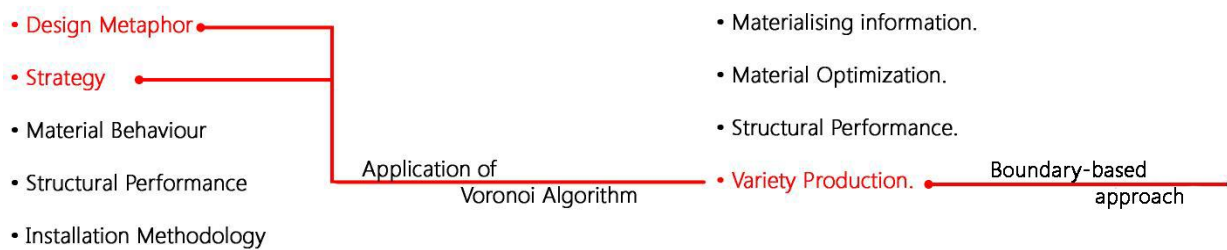
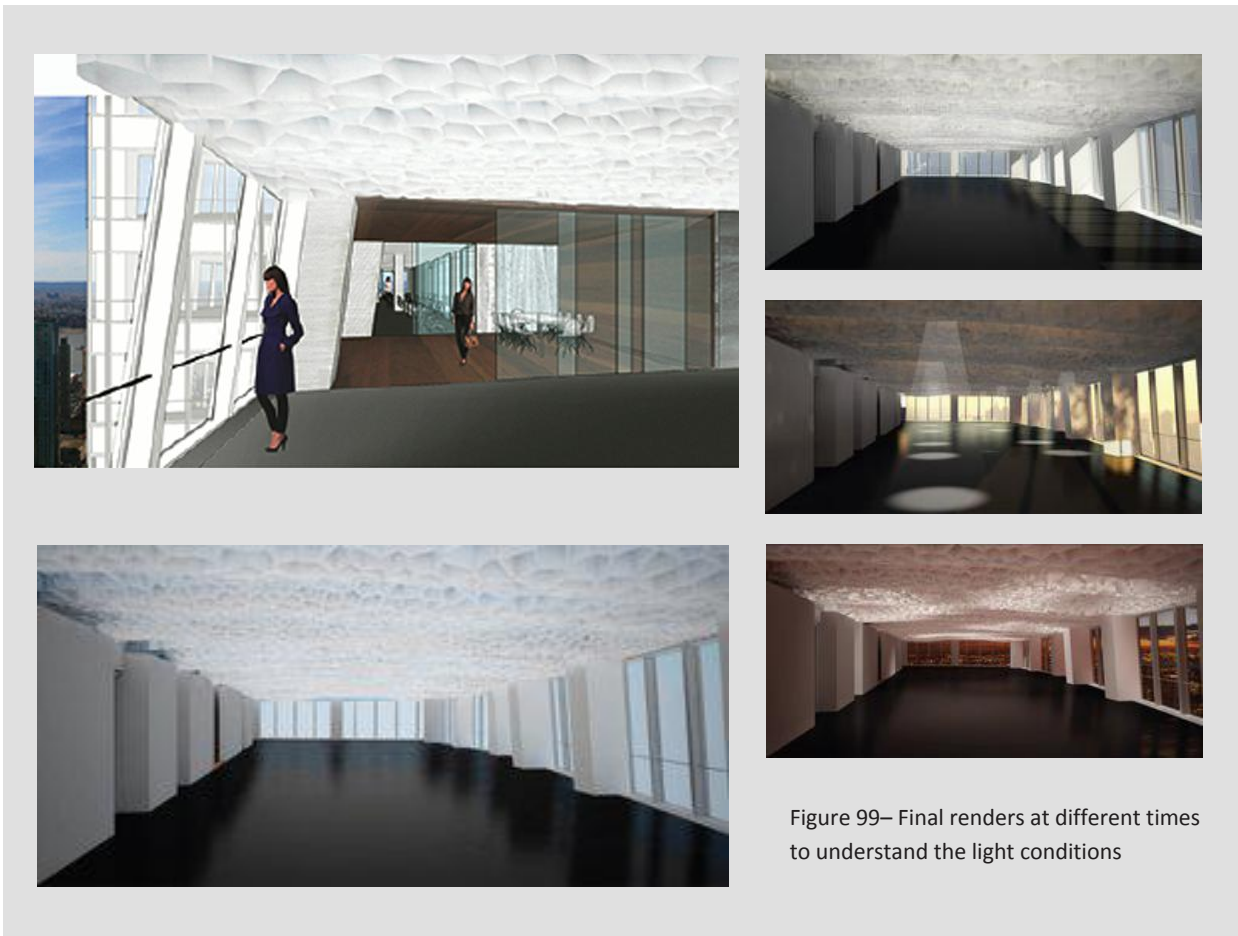


Figure 97 – Internal framing



Figure 98 –



Chapter One. Introduction-Voronoi Geometry	
A Brief History 2D Voronoi Centroidal Voronoi Tessellation 3d Voronoi Other types of Voronoi	Chapter two. Voronoi Geometry in nature Dragonfly wing Soap Bubbles
Chapter three Understanding Algorithms as a tool in Design	
Algorithmic Techniques Spiralling Packing Weaving Blending	
Chapter four Tessellation/Tiling Cracking Flocking	Understanding Voronoi algorithm as a tool for tessellation
	Case Studies.
	Chapter five - Dragonfly installation
	Chapter six - Water Cube Pavillion
	Chapter seven Voronoi Algorithm in interior scale Case Studies. C_Wall Smart Cloud Vessel Chair Paradise Pavillion
	Chapter eight Investigation into Digital methods and manufacturing
	Chapter nine - Conclusion

Vessel Chair

Design Metaphor

The design of the Vessel chair is driven by the Voronoi tessellation pattern.

It is seen in bone, turtle shell structure, giraffe pattern; nature's solutions for structural optimization.

Strategy

Grasshopper plugin for Rhinoceros3D modelling platform is utilized in order to model the parametrically designed frame;

The vessel chair is designed as an responsive mass-customized furniture. Designed in Grasshopper/Rhino computer platform the structure is scripted around 3 variables:

- a parametric algorithm to reduce material/mass by design through deduction.
- Patterns and algorithmic geometries that eliminate unnecessary mass.
- Patterns and algorithmic geometries for organic optimization.

The users

- Height
- Weight
- A raster image

Based on these parameters, the algorithm recalculates the amount of structure needed according to the height/weight ratio of the user and takes away the unnecessary material, changes the proportions to the most optimum dimensions. The third variable, the users submitted picture, is to be analysed for the colour selection of the chair

If the user is happy with the outcome the design could be submitted [baked] to be processed as CAM [Computer aided manufacturing information for CNC milling.

The Vessel Chair is a part of Parachair collection by fixhtnk.



Figure 100 – Image of Vessel chair.

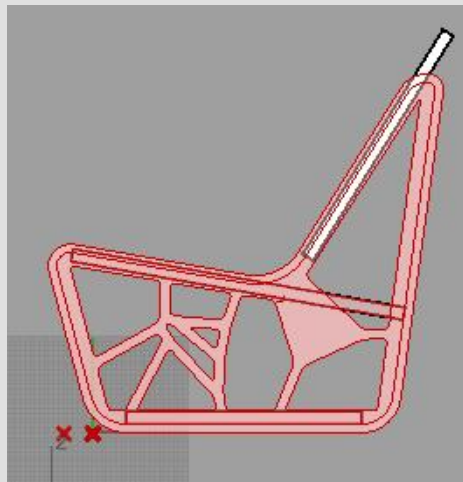
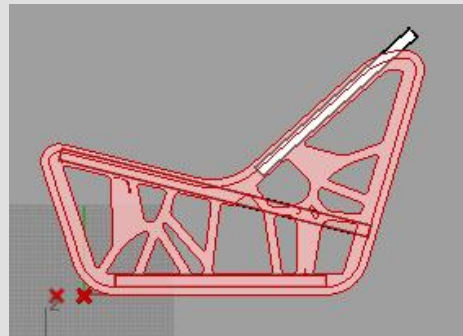


Figure 101 – CG Image in rhinoceros software to show different configurations.

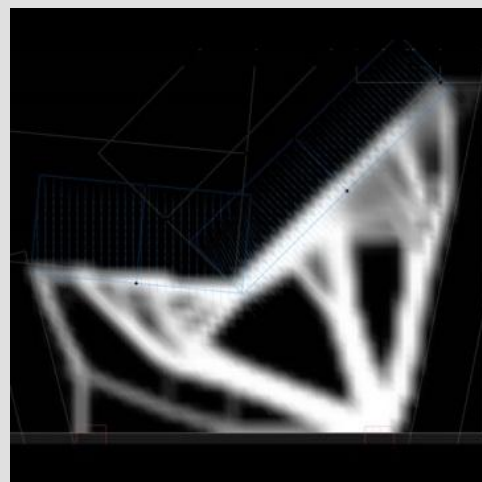


Figure 102 -

Material Behaviour

The chair frame is constructed with one inch plywood, milled with 3axis CNC machine.

The seat surface is created by extruding the edges of the structure cut-outs of the frame. The material is flexible polyester resin fiberglass

. The internal structure of the material and the perpendicular organization of the core structure of the bamboo-ply had a much better performance than a typical plywood structure. The connection pieces being out of resin had a different behaviour which was not in scope of research for the team.

Structural Performance

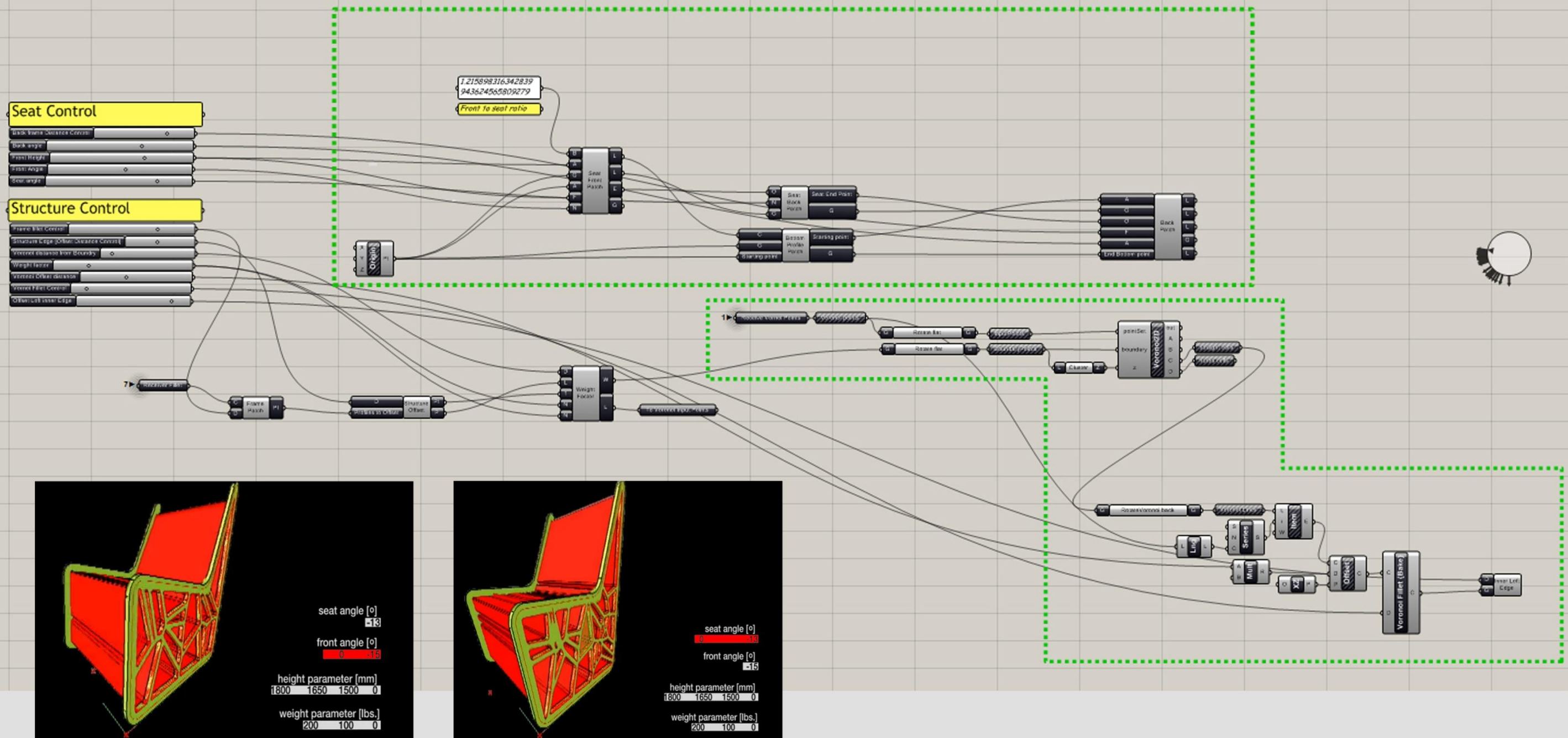
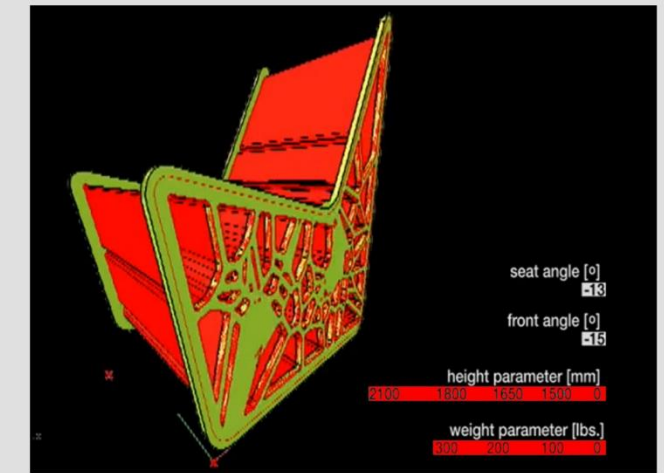
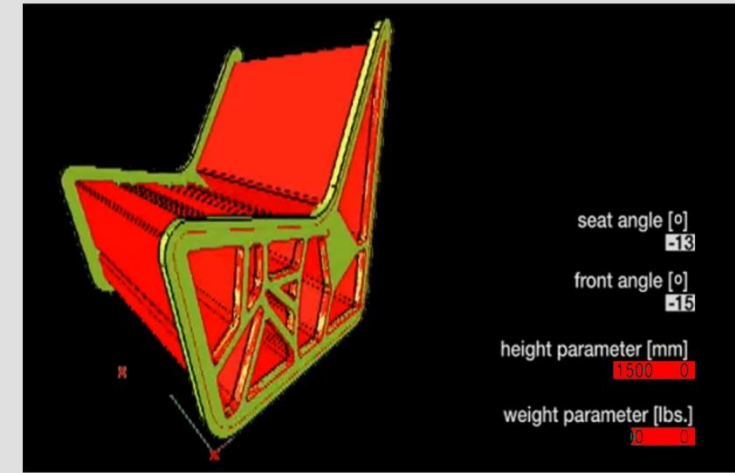
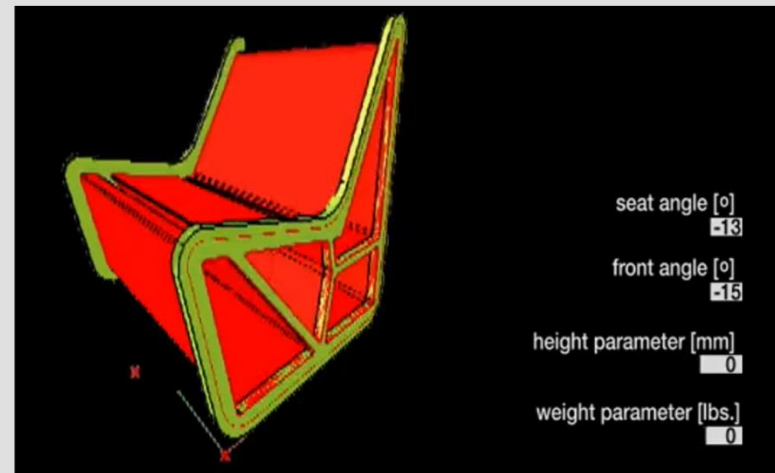
With using Voronoi as a "mass-deducting algorithm" the structural performance on the material is tested at first and then customizing the pattern attributes and attraction points the strategy is reversed to use the algorithm to allow for less mass deduction and preserve the material for a better structural performance.

Installation Methodology

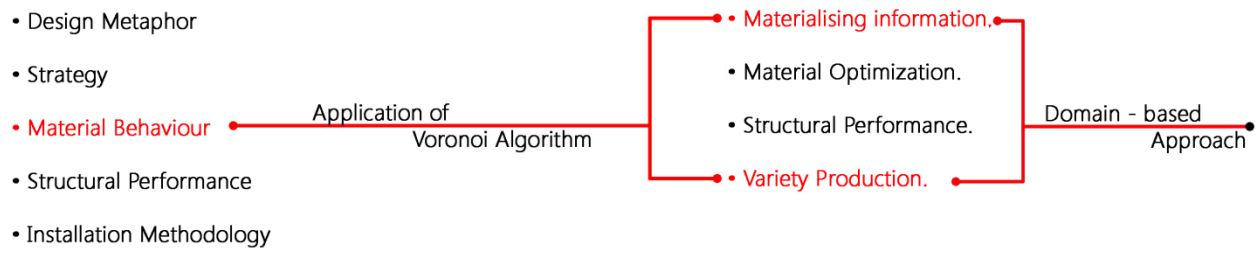
If the user is happy with the outcome the design could be submitted [baked] to be processed as CAM [Computer aided manufacturing information for CNC milling

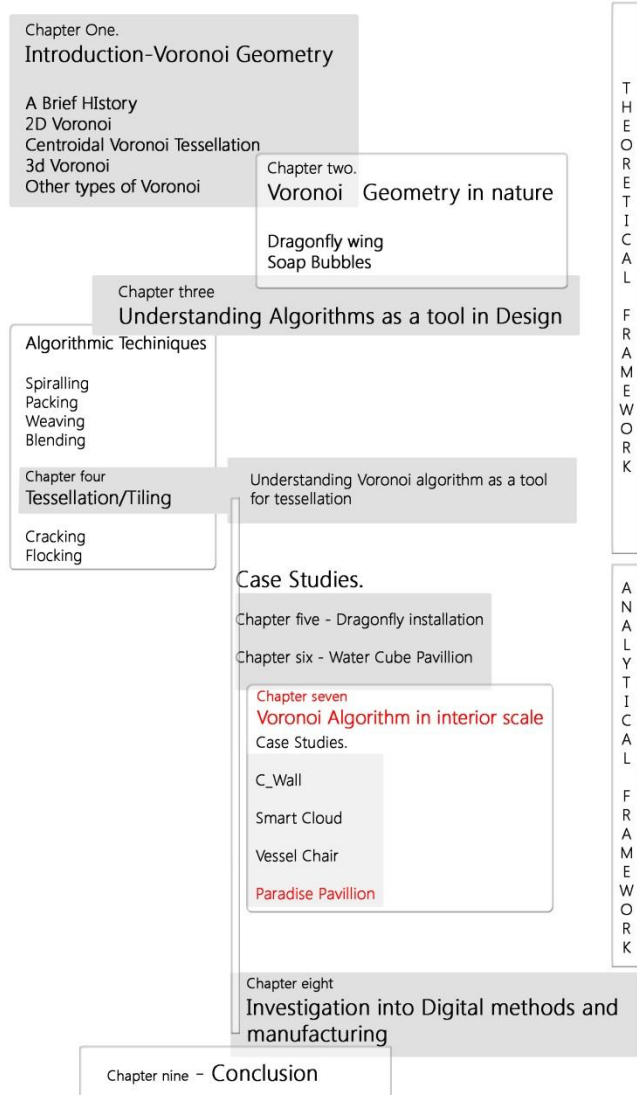
The vessel chair is designed as a mass-customized piece, which means all joints and connections need to be modular and interchangeable. The approach was to simplify the connections. Parts are milled with a 3 axis CNC and the flexible parts are laser-cut flat, and layered with plasticized-resin on fibre glass to connect the two sides.

.



Thus, in this project the algorithm is mainly used for variety production, the controlling variable in it was of material optimization.





PARADISE PAVILION

- Entrant:
Taiyo Membrane Corp
- Client:
Chris Bosse
- Architect:
Taiyo Membrane Corp & Chris Bosse
- Structural Engineer:
Xiang Du (TMC)
- Fabricator(s):
Taiyo Membrane Corp
- Building Materials:
Specially treated high-tech Nylon and light
- Dimensions:
7x7x7m
49 sqm (quadrat meter) 350 cbm (cubic meter)
- Weight:
17 kg.
- Construction/manufacturing time:
4 weeks.

Design Metaphor

Microscopic cell structures served as the inspiration for the design of a pavilion that is reminiscent of irregular natural forms like foam, sponge, or coral reefs.

Chris Bosse from PTW Architects, co-responsible for giving 'The Watercube,, Beijing's National Swimming Centre its soap bubble structure, created these biomorphic shapes using architecture software.⁵⁶

The phenomenology and structure of micro-organisms like coral polyps or radiolarians are the basis of the computer simulation of naturally evolving systems.



Figure 103 – An image of paradise pavilion



Figure 104 – Paradise Pavilion in different light to change the ambience



Figure 105 – Different kind of cells presenting varied type of volumes.

⁵⁶ <http://www.architter.com/fullScreen.php?id=717&img=1>

The shape of the pavilion is not explicitly "designed", it is rather the result of the most efficient subdivision of three-dimensional space, found in nature, such as organic cells, mineral crystals and the natural formation of soap bubbles. (Voronoi algorithm)

This concept was achieved with a flexible material that follows the forces of gravity, tension and growth, similar to a spider web or a coral reef.

Structural Performance

Structure and Space

The project renounces on the application of a structure in the traditional sense. Instead, the space is filled with a 3-dimensional lightweight-sculpture, solely based on minimal surface tension, freely stretching between wall and ceiling and floor.

Innovation and Digital Workflow:

The product shows a new way of digital workflow, enabling the generation of space out of a lightweight material in an extremely short time. The computer-model, based on the simulation of complexity in naturally evolving systems, feeds directly into a production-line of sail-making-software and digital manufacturing.

Transport and Sustainability

The pavilion (weight: 17 kg) is transportable in a sports-bag to any place in the world; can be assembled in less than one hour, and is fully reusable.

While appearing solid, the structure is soft and flexible and creates highly unusual spaces which come to life with projection and lighting. Projects of any scale and purpose can be realized in a short amount of time2.



Figure 106 and 107 – Images of paradise pavilion

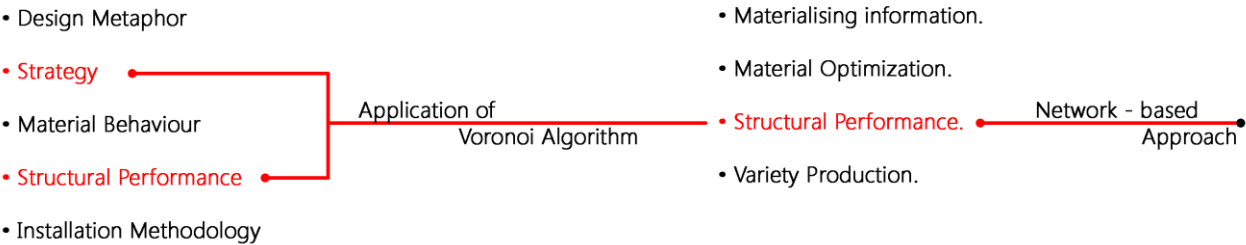
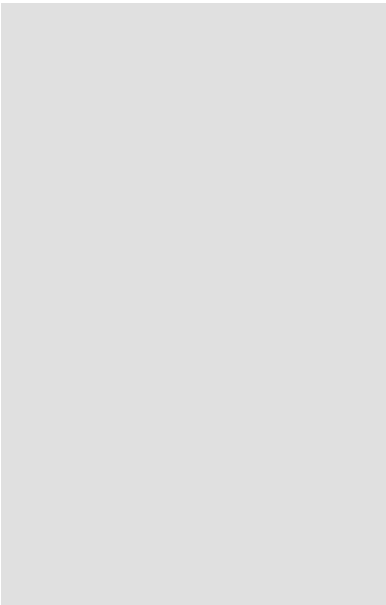
Minimal Surfaces

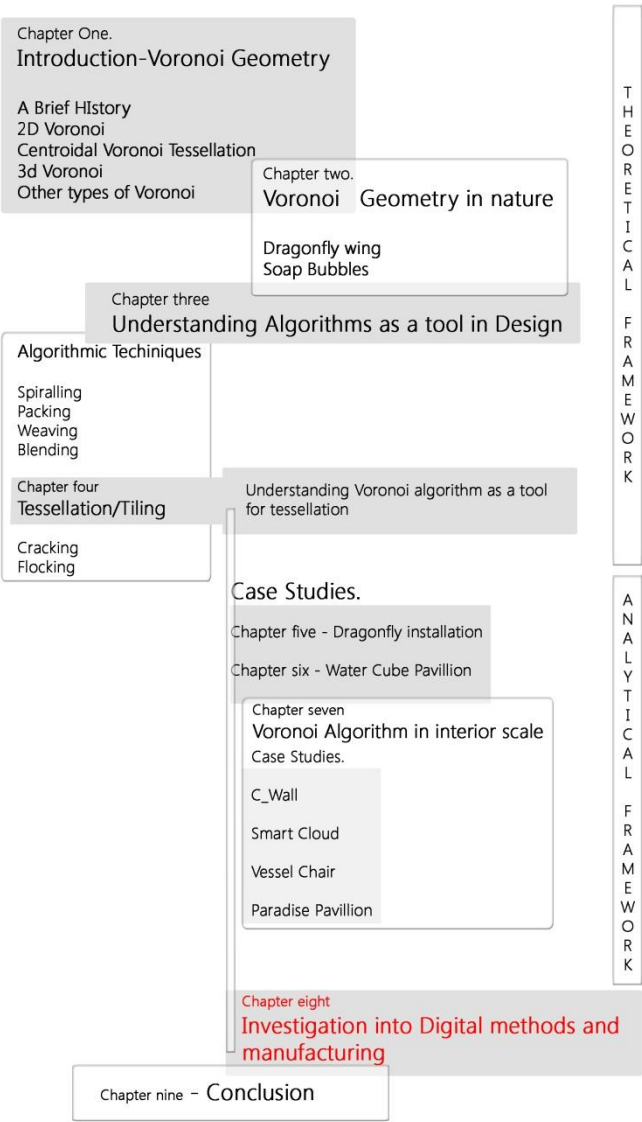
(-any surface that has a mean curvature of zero. - for a given boundary a minimal surface cannot be changed without increasing the area of the surface-).

The lightweight-fabric-construction of the pavilion follows the lines and surface-tension of soap films, stretching between ground and sky.

These natural curves of bubbles are translated into an organic 3 dimensional space.

Since the early seventies, with Frei Otto`s soap-bubble experiments for the Munich Olympic Stadium, naturally evolving systems haven`t lost their fascination in the field of new building typologies and structures.





Today computers are used at every step of design process, from conceptual design to construction. Three-dimensional modelling and visualization, generative form finding, scripted modulation systems, structural and thermal analysis, project management and co-ordination, and file-to-factory production are just some of the digital practices employed by architects and building consultants. Digital fabrication is often one of the final stages of this process, and it is very much what it sounds like; a way of making that uses digital data to control a fabrication process. Falling under the umbrella of computer-aided design and manufacturing (CAD/CAM), it relies on computer driven machine tools to build or cut parts.

Many of today's computationally based design approaches to complex geometric forms focus on arbitrary form generation, with minimal attention paid to manufacturing construction and structural efficiency.⁵⁷

Primary Structural and Construction Specific Considerations

Construction Considerations

Historically speaking, when geometrically complex building forms were built, as with the works of Victor Horta for example, they respected the limitations of the current construction technology. The designer recognized their responsibility for expressing their design intent through precise and comprehensible representations that could be understood by all of the parties involved in the project. Even designers seeking to create apparently nondefinable forms began to develop new ways in which to manufacture the complex geometrical forms in line with the appropriate construction techniques.

Between the period of 1914 to 1926 when Antoni Gaudi worked on the Sagrada Familia, he developed a set of construction rules that the masons were able to follow. His generation of the principal architectural elements was based on "ruled surfaces" which included the hyperbolic paraboloid and the hyperboloid of revolution, both of which are doubly curved and non-developable.

While different in their architectural expression, the later works of Felix Candela and Pier Luigi Nervi used the same conceptual approach as Gaudi. These men made extensive use of those kinds of surfaces in the reinforced concrete structures that they designed. In this manner the wooden formwork could be easily erected out of flat wood planks.

(Schodek 2005, p49)



Figure 108 - Sagrada Familia, designed by Antoni Gaudi, period of 1914 to 1926.

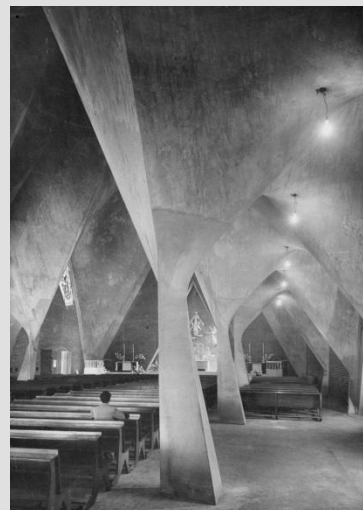


Figure 109 - Church of La Virgen Milagrosa, Mexico City, designed by Felix Candela, built 1953–55..



Figure 110 - Copertura-palazzetto dello-sport-roma, designed by Pier Luigi Nervi

⁵⁷ From control to design.

Structural Considerations

Structural efficiency is an aspect of design that may or may not be explicitly considered when generating complex building forms. While many civil engineering structures that utilize complex geometries (dams) are responsive to both structural and technical efficiency, this is often not the case with regard to architectural constructions. The simple act of forming a curved surface does not automatically infuse it with the positive structural benefits that are possible with certain curved surfaces. The classic doubly curved shapes such as portions of spheres or the hyperbolic paraboloid shapes used by architects in the late 19th and early 20th century have been widely proven to demonstrate “*membrane action*” where internal forces are efficiently transmitted through the surface of the shell in an in-plane manner. See Figure()

When this scenario exists, the stresses acting out of plane within the surface are quite low and thus the shell can be made quite thin. Membrane action does not exist in all curved surfaces and its presence in a surface depends on the existence of particular combinations of surface shapes and types of loading conditions. It is important to note that with a corresponding decrease in the amount of material associated with the proper development of a structural skin that exhibits membrane action the skin will also be more susceptible to deformation due to local or point loads. A proper balance between these must be met or the design of the membrane must act on a variety of levels to redistribute stresses imposed on it.

The misconception that curvature automatically translates into structural efficiency is quite prevalent in construction today. Complexly curved surfaces and their widespread use can often be immature versions of properly designed surfaces that could potentially exhibit the desired characteristics of membrane action. It is only through careful examination of the design; functional criteria and intent along with structural analysis can the final product exhibit the structural advantages associated with a curved surface. (Schodek 2005, p48)

Digital Form-Generation Techniques and Shape Generation

Common vs Uncommon Approaches.

Common – The Designs are envisioned by the user and the digital tools act to develop and represent these ideas. The inspiration for complex and unique forms is derived from many different sources, ranging from direct responses to programmatic requirements.

Uncommon – The designers develop computational environments whereby the design is developed by the program through pre-specified rule structures or other principles.



Figure 111- Roof of Nervi's Palazzetto dello Sport which exhibits membrane action

The most widely used approach for the form generation used by designers is the direct use and manipulation of computational tools (points, lines, splines, lofts, sweeps etc.) commonly found in a variety of digital modelling environments (form-Z, Rhinoceros, MicroStation, etc.)

Computational tools that are visually oriented and based on descriptive geometry or on other mathematical means of describing lines, curves, and surfaces can also be used in a more direct manipulation process to generate forms. Software technologies associated with this type of shape derivation are uncommon in the architectural design environment but are found in broad based mathematical tools (MathCAD, Mathematica, Maple).

In an effort to derive forms based on a set of external influences, be they real or metaphorical, some designers have adopted the use of software (Maya) that allows for an influence of form based on force functions of one type or another. Objects or functions within an environment can be given a defined set of controllable parameters that afford them the ability to influence and interact with other objects that can in turn push, pull, deform and essentially drive shape generation for the resultant form.

Parametrically driven shape derivation is also being used in a more controlled manner, whereby the forms are generated according to sets of predefined rule structures and component parts. The design approach within these software applications can vary from one to another where priority can be placed on having a strong construction rationale or through different programmatic or conceptual intents (Generative Components, CATIA, SolidWorks, Unigraphics, CADD5). A commonly used approach here is to define a set of parameters for a structural element whose form drives the formation of the building envelope. The parameters defined can be related to the physical dimensioning of a component or any number of relevant values or relationships. Through direct manipulation of these control parameters the changes will propagate throughout the model to instantaneously update it.

A recent trend is based on an approach that seeks to derive form through the implementation of genetic growth or repetition algorithms. Patterns seen in nature such as fractals, tessellations and Voronoi can be broken down into complex rule structures that can be in turn modified and used for shape generation.

The Whole and its Parts

All architectural structures – unless dug into the Earth – have this one thing in common; they are assemblies of numerous parts joined together. This statement is based upon the simple fact that no single material, be it natural or manufactured, is as large as a building. In architecture, the whole easily consists of thousands of parts. And if the architect used contemporary digital-modeling tools to generate a complex form, each element of the design is probably different.

In today's design process. The introduction of digital design and fabrication tools leads to a continuous flow of information from conception into production. In the architect's practice, CAD software helps to define surfaces by pulling control points or adding and subtracting volumes like a virtual sculptor. Via standard data formats the design is forwarded to CAM (Computer Aided Manufacturing) in the workshop, which in turn controls the computerized tools that produce the various components. Does 'CAD/CAM' mean that architectural design is reduced to pulling control points, exporting the result and leaning back to watch the production?

This method indeed is applicable to small-scale prototype design or mold fabrication where the whole consists of just one part. In architecture however, the whole and its parts are not identical: CAD modelling tools help to define the whole, while a CNC machine fabricates parts. And even worse, the building material at disposal within the budget often does not match the desired shape: plain sheets and straight bars and profiles confront curvilinear designs. How to translate the shape of the whole into parts made from standardized material? Here in fact, between the modelling tool and the fabrication tool, lies the complete design process including the breakdown into parts, the optimization according to various constraints, the detailing, and the preparation for fabrication. There is neither a generic software tool to automate this process from the whole to its parts. Building designs are too different for a generally applicable solution. Nor is the manual processing of thousands of different parts advisable, because it is laborious and hard to get a grip on it. Approaching a digital architectural planning process means establishing project-specific algorithmic relations between whole and its parts.⁵⁸ This approach can be categorized into these steps.

- Organize the relation between whole and its parts:
Parametric 3D CAD Models
- Optimize the whole depending on the interrelations among its parts:
Algorithmic optimization tools
- Simplify the Parts
Rationalizing parts to realize the design
- Materialize information
Production Data for the parts

⁵⁸ From control to design.

Organise the relation between the whole and its parts

Implementation of customized parametric models on the basis of professional CAD packages is done to organise dynamically the breakdown from the whole to the

parts.

Complex Units are spatially difficult to comprehend.

For example the intertwining forms on UNStudio's Mercedes-Benz Museum can hardly be described in plans and sections.

However contractors needed exact documents with information about the parts while the whole was still in process of development. By organizing the immense amount of information in a

parametric 3-dimensional model a couple of thousand detailed plans could be generated automatically for any variant of the design chosen by the architect.

Designers don't think in numbers. They think in relations. Standard CAD systems don't store relations. They store numbers. Numbers that change while their relations remain stable. Parametric CAD models capture those persistent rules behind the developing form, reducing thousands of co-ordinates to a handful of parameters. The consequence of describing thousands of different parts with a few parameters is a new kind of 'parametric standardization'.

Individuality is expressed in variables.



Figure 112 – Mercedes Benz Museum

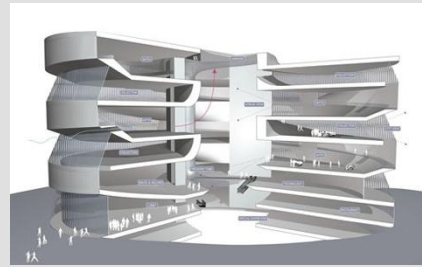


Figure 113 – Mercedes Benz Museum
Sectional perspective

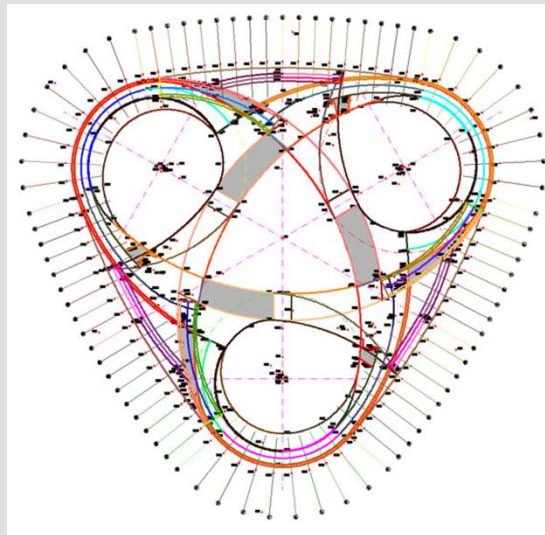


Figure 114 – Mercedes Benz Museum
Plan

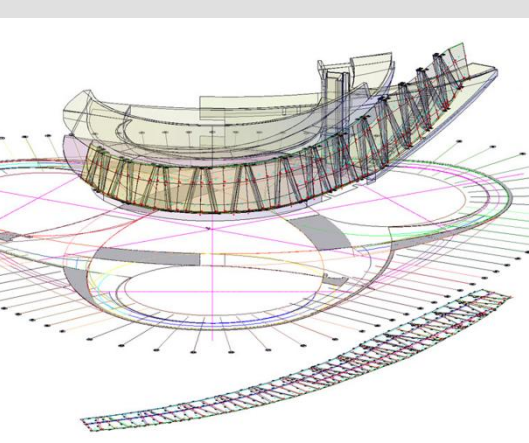
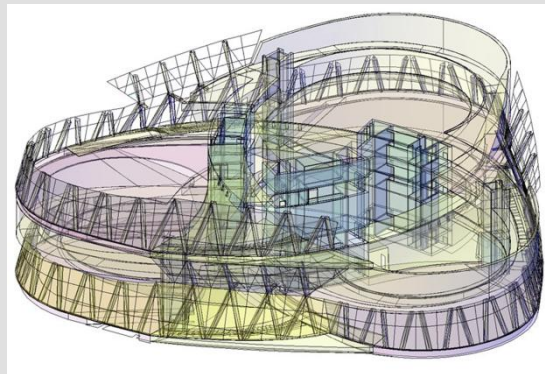


Figure 115 – The glass panels of the façade were generated

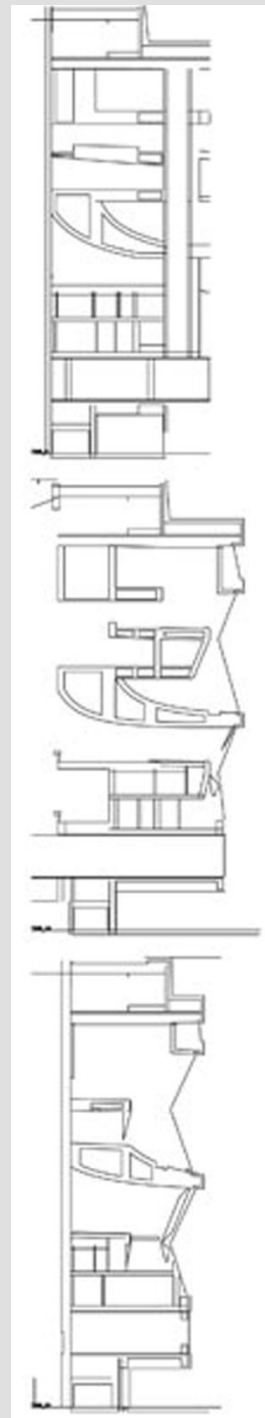


Figure 116 – Cross
Sections

Optimize the whole depending on the interrelations among its parts

There has to be a relation among the parts to develop advanced optimization tools that match design ideas to the best constructive, structural and functional solution.

The leaning column in the basement of KCAP's Groningen Stadsbalkon have to hold up a 3000sqm concrete slab, get out of the way, minimize resources, and give the impression of a random forest. Such conflicting demands are hard to tackle with a systematic approach.

With an optimization software based on artificial-life methods the architects were enabled to "grow" alternative solutions on their screens that matched all constrains – far quicker than a real forest.

In a rectangular design with regular grids and uniform modules the optimum is easy to determine. But in a complex geometry, where dimensions change and angles vary, it can be difficult to find any answer at all – let alone a good one. With state-of-the-art optimization tools that exploit the power of bottom-up methods like Genetic Algorithms and Swarm Intelligence it is possible to find good solutions for complex systems – maintaining the non-regular form instead of falling back on a square grid.

Simplify the parts

Architectural construction is all about assembly of parts. And complex architecture consists of large number of individual parts. For example a little extruded dovetail profile was used by a firm called DESIGNTOPRODUCTION in several constructions. While the little profile is a standard invariable industrial product, the grooved counterpart of the component is a parametric detail. More than three thousands of them are needed to connect the 2164 pieces of Daniel Libeskind's sculpture Futuropolis. And besides looking good each of them saved a few minutes of labour time, because they make it unnecessary to clamp the pieces during the hardening of the glue.

Integrating thorough knowledge about fabrication technologies, materials and joints into the detailing leads to smarter, leaner and more rational production processes – and to a result that comes close to the intention of the original design without busting the budget.

Most of the conditions of breaking down the whole into parts are determined by machine's dimensions, its tools and scope of movement.



Figure 117 –
Groningen
Stadium

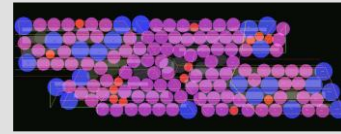


Figure 118 -
Twister in
action, color
coding by
column type

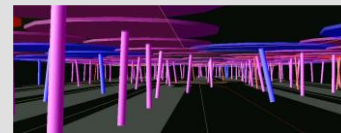


Figure 119 -
Color coding by
kinetic energy

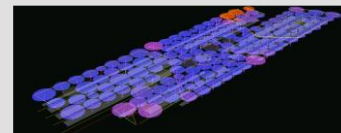


Figure 120 -
Color coding by
excessive tilt
angle.

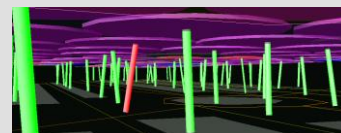


Figure 121 -
Libeskind's
Futuropolis.

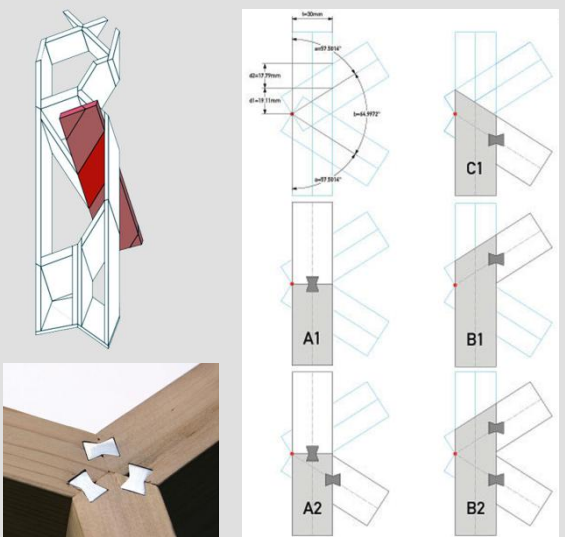


Figure 223 -
Details of the structure

Figure 122 – The dovetail connector

Materialize information

Non-standard geometries are built from non-standard parts. In a workshop every single part has to be edited for the computer-aided machine- nesting parts on raw material, selecting tools, configuring the tool path and generating the machine code.

For example the doubly curved panels on Zaha Hadid's Hungerburg Funicular Stations are held in place by some 2500 individually shaped profiles. Each of them are cut from polyethylene boards with a computer controlled five – axis router. Manually nesting a couple of thousand pieces and translating their geometry into NC-programs for the router would have been a heavy burden for any building budget. Herefore the complete machine code was directly generated from a parametric 3D – model including stickers with unique part IDs that help allocate the pieces. While software solutions offer to perform many of those steps automatically, still every single part has to be imported and treated with CAM software tools – an enduring

process that has to be repeated any time a condition changes. Automating the planning from detailing to machine code is the final step of organising the relations between the whole and its parts, but is adding most value to the processing chain⁵⁹.



Figure 123 – Hungerburg Funicular



Figure 124 – Assembly parts

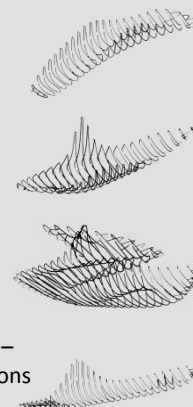


Figure 125 –
Cross sections

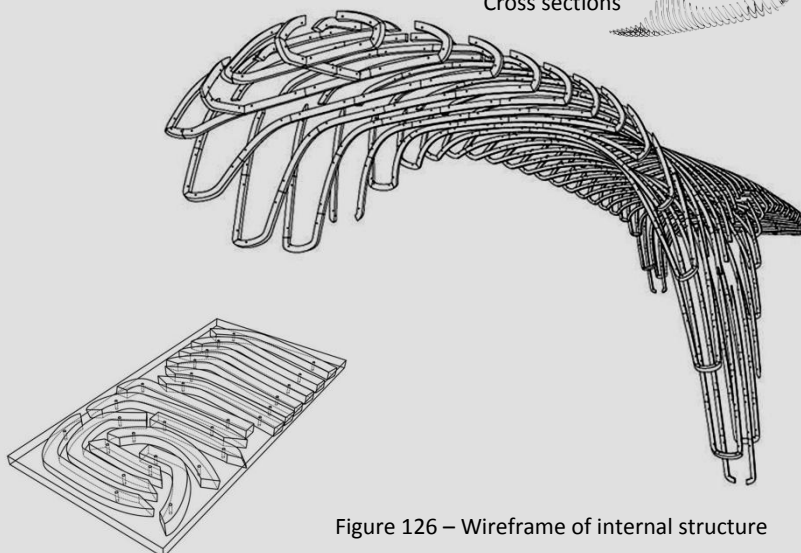


Figure 126 – Wireframe of internal structure



Figure 127 –internal structure

⁵⁹ From Control to Design – Parametric/Algorithmic Architecture.

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Dragonfly wing Soap Bubbles	
Chapter three Understanding Algorithms as a tool in Design	
Algorithmic Techniques	
Spiralling Packing Weaving Blending	
Chapter four Tessellation/Tiling	
Cracking Flocking	
Understanding Voronoi algorithm as a tool for tessellation	
Case Studies.	
Chapter five - Dragonfly installation	
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Conclusion

Voronoi Geometry

"It's not just an accumulation of aggregates of the volumes but that every time that there is a subdivision many many times more polymorphic or more kind of landscaped and aggregation of functions or forms". –

Cristos Passas

Defining Algotecture

Algotecture is a term coined for Algorithmic Architecture.

This term differs from the popular terms *CAD* or *computer graphics* in the sense that algorithms are not necessarily dependent on computers whereas the former are, at least, by definition. This distinction is very important as it liberates, excludes, and disassociates the mathematical and logical processes used for addressing a problem from the machine that facilitates the implementation of those processes. Such a use involves the articulation of a strategy for solving problems whose target is known, as well as to address problems whose target cannot be defined.

The nature of algorithms: *"algorithms are a design medium. Algorithmic design is simply the systematic encoding of a design process, often into a programming language, many people view computation as more of a technical skill than a creative one. The consequence is that some people view it as out of their grasp and completely incomprehensible, much in the same way some approach computers. Others have perhaps the opposite reaction. They see it as so technical, it does not count as art. They believe it is somehow outside the designer's hands. A technical process is perceived as something almost platonic, as though it has an objective existence that the designer is merely employing. This perspective is highly limiting. Computation is like any other medium. It has its strengths and weaknesses that effect how you approach a particular design problem"*

Jesse Louis-Rosenberg. Attended MIT, majoring in Mathematics. He previously worked as a consultant for Gehry Technologies in building modelling and design automation.

Computation

Design process often gets defined many ways, every individual has a different approach to inquire, observe and react. But for all approaches there is one thing which is in common and that is logical and step-wise thinking and decision making.

This act is in a way computation in human mind.

Algorithms

A tool to perform.

Algorithms are important in design as it can be seen as a set of procedures that lead stochastically towards the accomplishment of a goal.

Algorithms have a self-referential property. Algorithms can generate other algorithms; not only precise, identical, multiple copies of themselves but also structured text(i.e. code). That when executed will behave as an algorithm. In fact the process of composing an algorithm is also an algorithm in itself, that is, the algorithm that created the algorithm.

Voronoi algorithm

Algorithms act as private gravity for a design process. It can be seen as a systematic approach to handle any design problem. Such implementation of algorithms gives multiple outputs with reference to many variables. It completely depends on the designer to choose the most appropriate output based on his defined variable.

Concluding When and why Voronoi:

As we earlier categorised the process of design into three simple categories, which are design programme, concept and context, Voronoi algorithms can be understood as a very basic tool that gives us a glimpse of computational benefits.

While Understanding such an approach in terms of computerization and computation, it helps a designer in many ways.

Computerization:

While taking such an approach in computerization it helps a lot into producing many options and basically creates a template which is very flexible in terms of iterations in design.

In a design process the output or design model is very available at all moments in terms of presentation as well as extracting drawings.

All intangible ideas and conceptual visuals can be easily given a mode of image. The restriction in terms of conveying thoughts and ideas can be easily avoided.

The aspect that makes Voronoi algorithm very useful under the relevance of computerization is that it always works in a very interlinked manner. If there is a very small change at any point while designing, there is no need to go back and start modifying everything from scratch, it automatically updates the changes and modifies the model accordingly.

Computation:

Voronoi algorithms when seen with a perspective of computation, it helps a lot while parameterizing design variables. It creates a very easy and accessible canvas in which every variable is accepted, be it tangible or intangible. The network opens up a very flexible system in which design variables can be added and deducted with ease and at any time.

Design Programme and Voronoi

Application for Voronoi Algorithm in any design is a very straight forward approach reflecting the design programme requirements. Reflection of programme can be understood in two different ways.

First when the output is conceived beforehand in terms of form, Voronoi algorithms help as this tool is easily flexible and adaptable.

Second way is that when design programme is generative or experimental. Under such conditions Voronoi algorithms makes it easy to link the variables and conceive form accordingly. In such case the output is purely a resultant of variables and the model is always open for modifications and iterations.

Concept and Voronoi:

In understanding Voronoi algorithm as an initiator of design process or a big idea, its acceptance or application is purely based on designer's sensitivity and contextual responses.

Voronoi algorithm can also be a very helpful approach in terms of bio understanding.

These algorithms can give a very experimental take on logical ideas and such process can be easily controlled and modified at any step.

As a problem solver the application is adaptable but it also is a lot controlled by the factor of scale. Its application can get limited at some situations where its scale will not give a satisfying result.

Context and Voronoi:

Voronoi algorithms have a very direct relation with the context. When the context for any design is providing a scope of its translation into variables, this algorithms works directly onto it to give a result. The translation of context when done in a logical method the resultant network can directly link them.

Now as the output of this algorithm is a network, it depends solely on the designer's sensibility on where and when to use such in their process. Voronoi algorithms can be looked upon as a final solution or as an intermediary step in any design process.

Thus **when and why Voronoi** should be purely based on the need and scope. Forcing such an approach on design can lead to various forms but the sensitivity and contextual relevance can go missing.

Computational approach does not end just with an output of form, but can help a lot more when dealt logically and in right place

Understanding Voronoi algorithm diagrammatically gives us three kinds of approaches

- Network based approach
- Domain based approach
- Boundary based approach

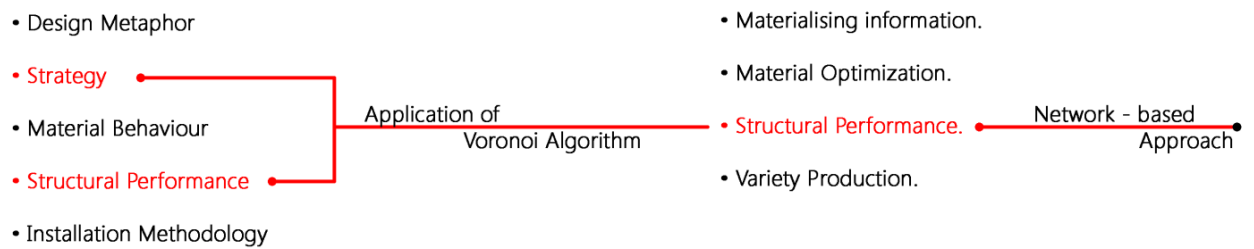
Network based approach	Boundary based approach	Domain based approach
Dragonfly installation	Water Cube Pavilion	Vessel Chair
Paradise Pavilion	Smart Cloud	C_Wall

As mentioned earlier all the defined categories for the analysis, in some way always respond to each other, they are all affected by the application of Voronoi algorithm.

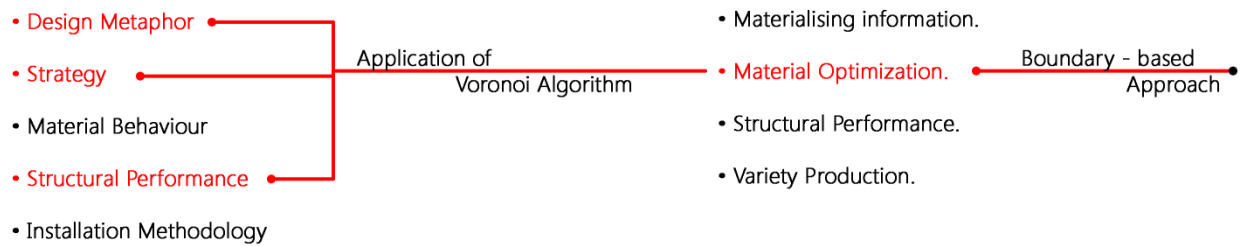
Chart explaining the effect of Voronoi Algorithm for all defined Categories.

	Design Metaphor	Strategy	Material Behaviour	Structural Performance	Installation Methodology
Dragonfly installation	●	● ●	● ●	● ● ●	● ●
Water Cube Pavilion	● ● ●	● ● ●	●	● ●	●
Vessel Chair	●	● ●	● ●	●	● ● ●
Paradise Pavilion	● ●	●	● ●	● ● ●	●
Smart Cloud	● ● ●	● ●	●	●	●
C_Wall	●	● ●	● ● ●	● ● ●	●

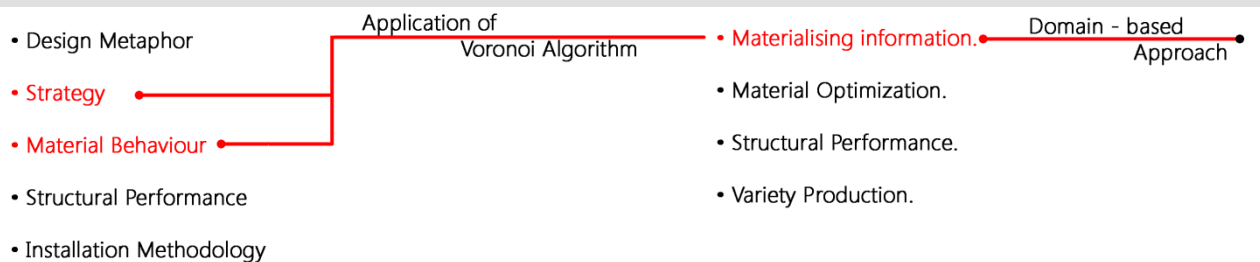
Dragonfly installation



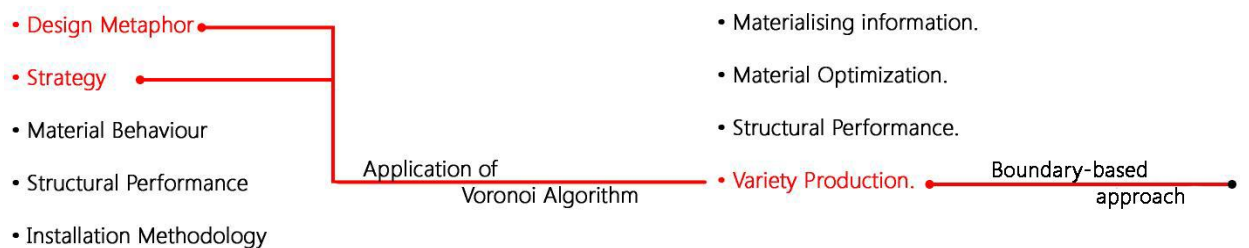
Water Cube Pavilion



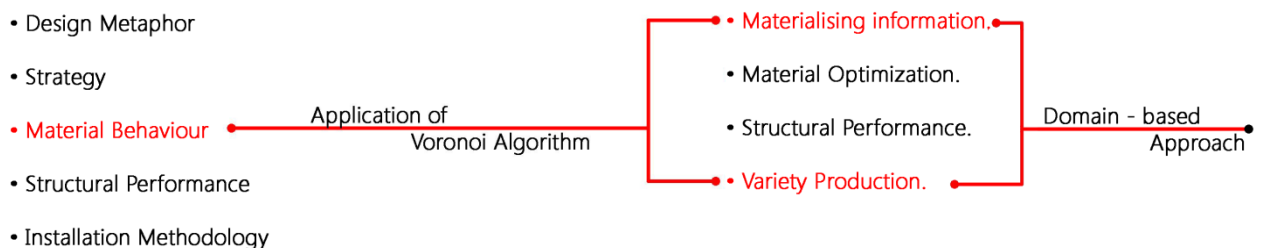
C_Wall



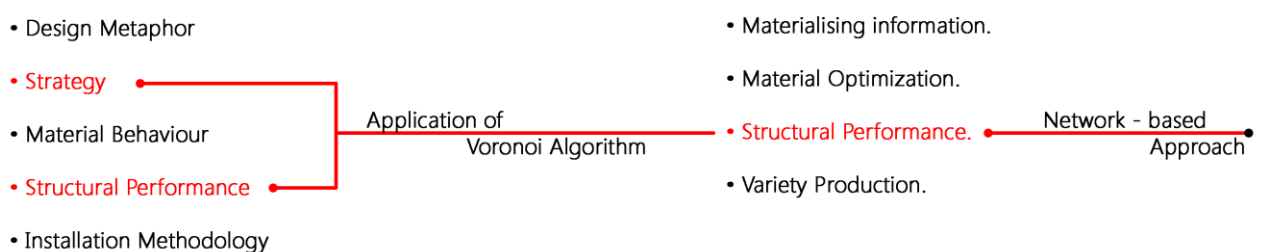
Smart Cloud



Vessel Chair



Paradise Pavilion



One of the primary constraints is that designing algorithmically requires manufacturing digitally, and there is a very limited set of tools currently available for that. There are also certain ideas that computation is good at exploring. While in most designing, a lot of the work goes into the direct effort of creation (painting, sculpting, drawing usually take a lot of physical time and effort), the advantage of a computational approach is that the creation time drops to nearly zero. This makes it a very good way to explore the concept of repetition because it is just as easy to make many things as one thing. It allows the possibility of infinite variability for the same reason. The ability to abstract your process and create a user interface also encourages interactivity and “customization.”

Generally algorithmic design is a very powerful medium, but working with it requires an understanding where its strengths lay.

FUTURE Research

A related and highly challenging topic for the future research is the computation of 4-D patterns in the form of animated facades and other flexible space structures. Ideas for such dynamic designs have been contributed by, for example. Kas Oosterhuis and his co-workers within the hyperbody research group at Delft University of Technology. These could provide the basis for further investigations from a mathematical/computational/engineering perspective to promote flexible architectural structures on a large scale.⁶⁰

The idea of time and temporality in architecture is often overlooked and it is in this regard that some architects (Kas Oosterhuis and Ole Bauman) have sought to develop buildings that effectively change throughout time and to various external forces. Here, architects are not designing static structures that maintain their structural form but ones that are capable of adapting to new uses or needs. Just as cultural changes occur over time, these buildings would modify their layout and organization to best serve the immediate needs of the user with the possibility to serve future uses equally well. Digital environments that support animation and motion (Maya) are useful here.

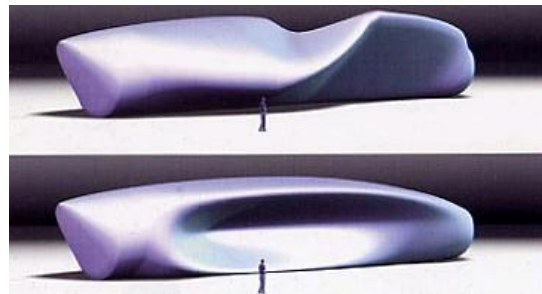


Figure 128 - Trans-ports by Kas Oosterhuis
Trans-ports is a data driven multimedia pavilion which can move like a bundle of muscles that are directed and it's conduct adjusted through the world wide web based game

⁶⁰ Architectural Design - Patterns of Architecture

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Figure 4 - <http://isnm.de/~schmidts/ucsb/voronoi/>

Figure 5 - Structural Analysis of a Dragonfly Wing by S.R. Jongerius & D. Lentink

Figure 6 - Structural Analysis of a Dragonfly Wing by S.R. Jongerius & D. Lentink

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